Error Estimation and Adaptive Mesh Refinement for Problems with Complicated Geometries

Marian Nemec

ELORET Corp.
Advanced Supercomputing Division
NASA Ames Research Center

Joint work with Michael Aftosmis (NASA Ames) and Marsha Berger (NYU)

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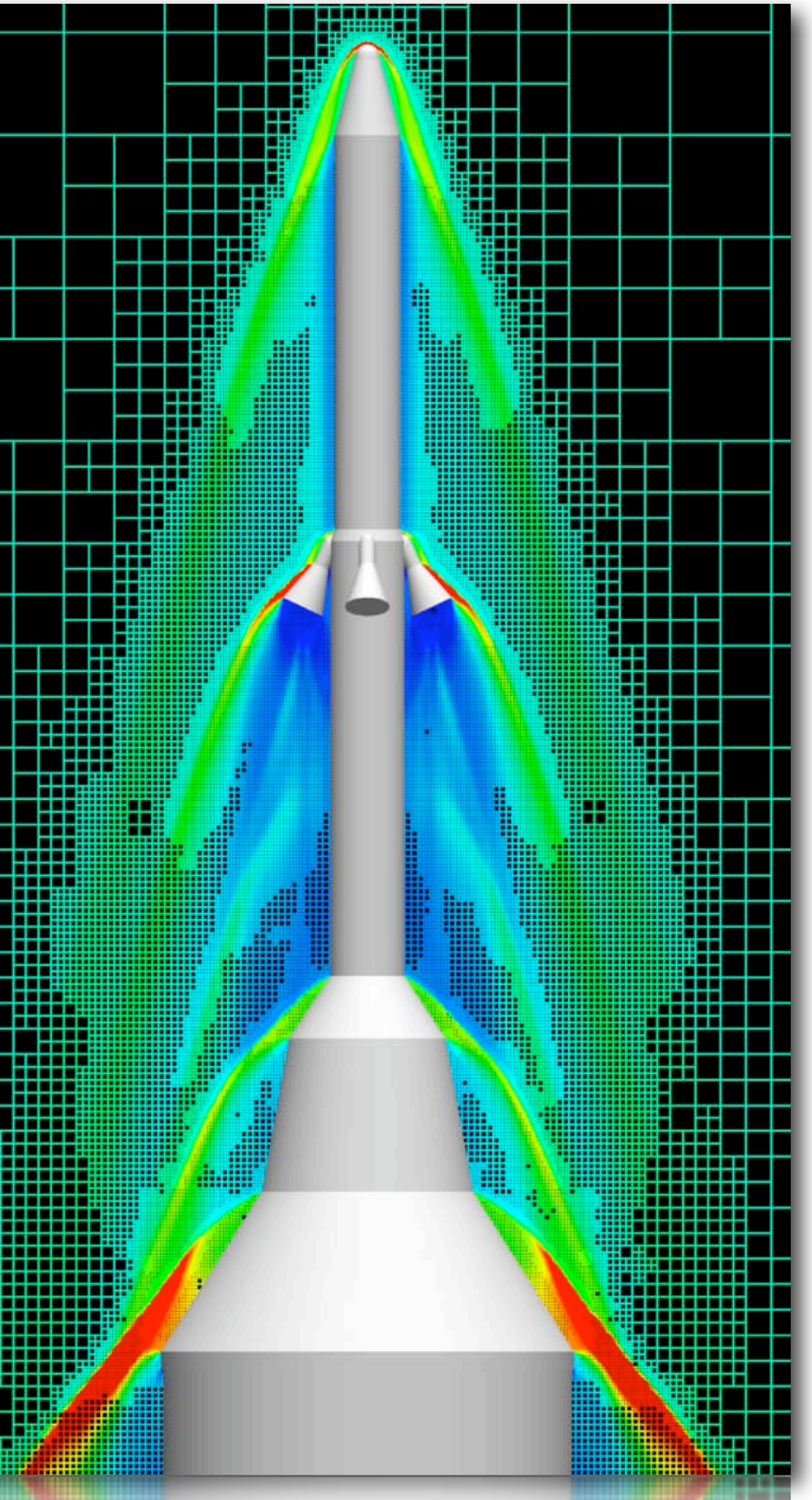
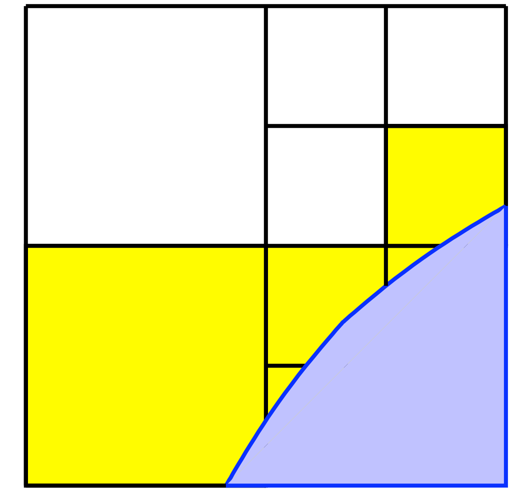
Objectives

Toward automation of CFD analysis

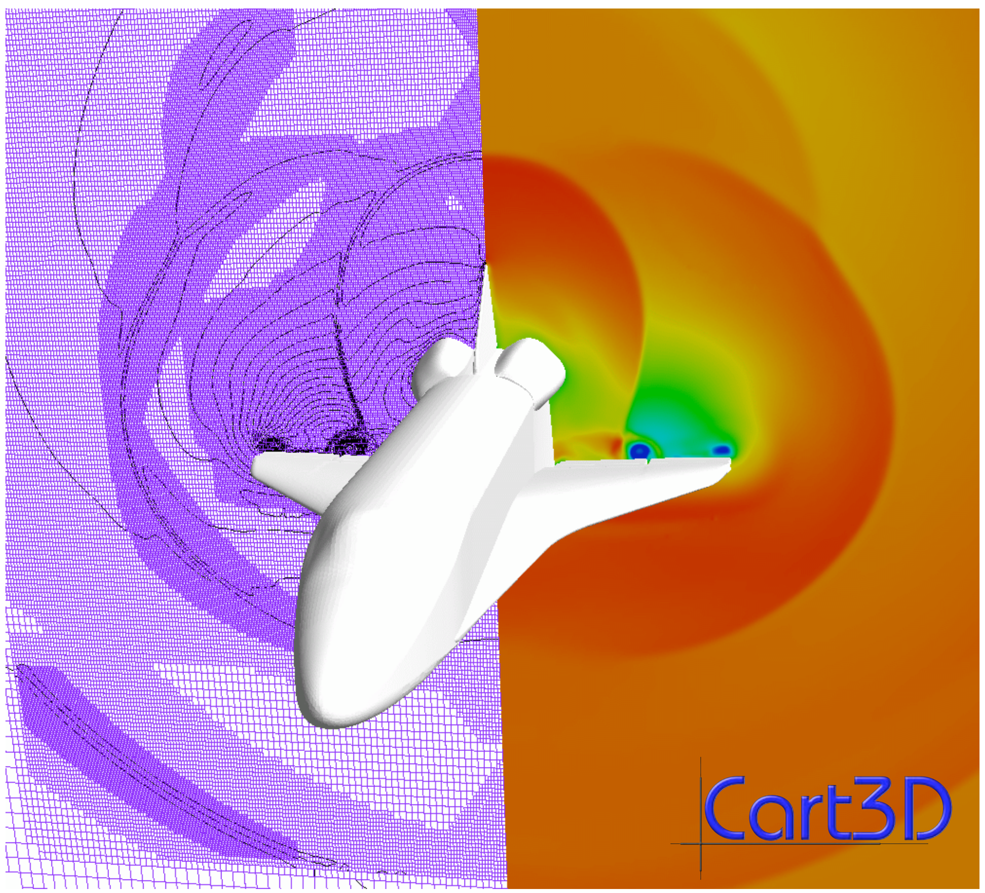


- Handle complex geometry problems
- Control discretization errors via solution-adaptive mesh refinement
- Focus on aerodynamic databases of parametric and optimization studies
  1. **Accuracy**: satisfy prescribed error bounds
  2. **Robustness** and **speed**: may require over 10^5 mesh generations
  3. **Automation**: avoid user supervision
- Obtain "expert meshes" independent of user skill
- Run every case adaptively in production settings

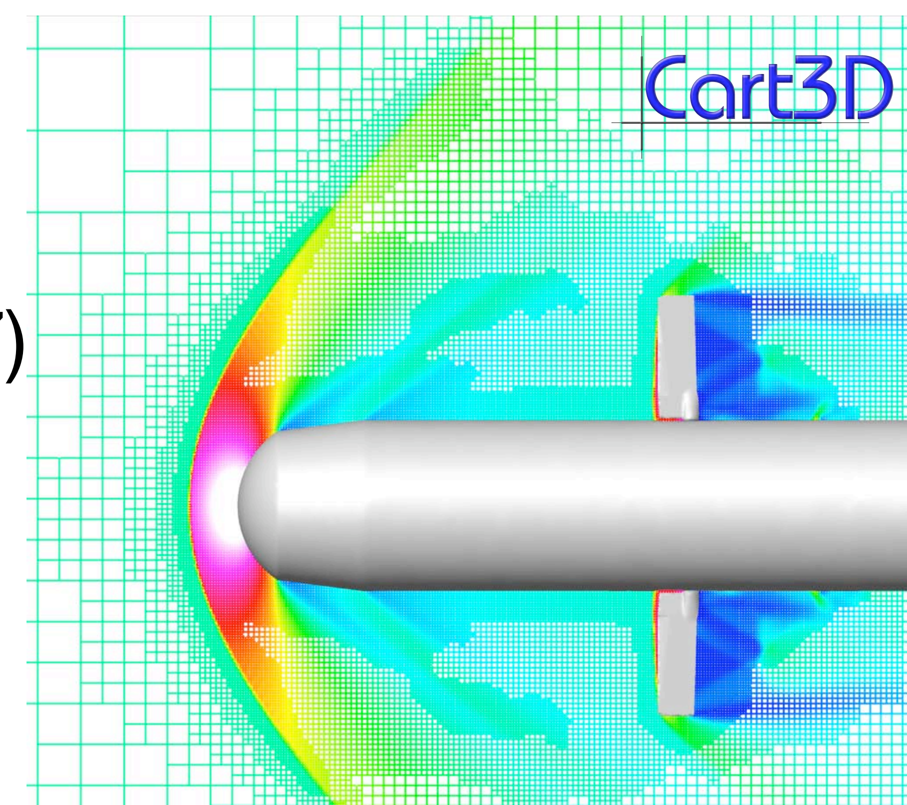
Approach



1. Embedded-boundary Cartesian mesh method (1990's)
   - Arbitrarily complex domains, efficient and accurate
   - Irregularity confined to body intersecting cells



2. Incremental strategy for h-refinement of nested Cartesian meshes (2002)
   - Fast local re-meshing of flagged cells
   - Guaranteed reliability
   - Early work used feature detection and τ-extrapolation



3. Adjoint-weighted residual error estimates (2007)
   - Mesh enrichment targets output functionals
   - Functional error-bound estimates
   - Implementation exploits nesting of Cartesian meshes for fast interpolation

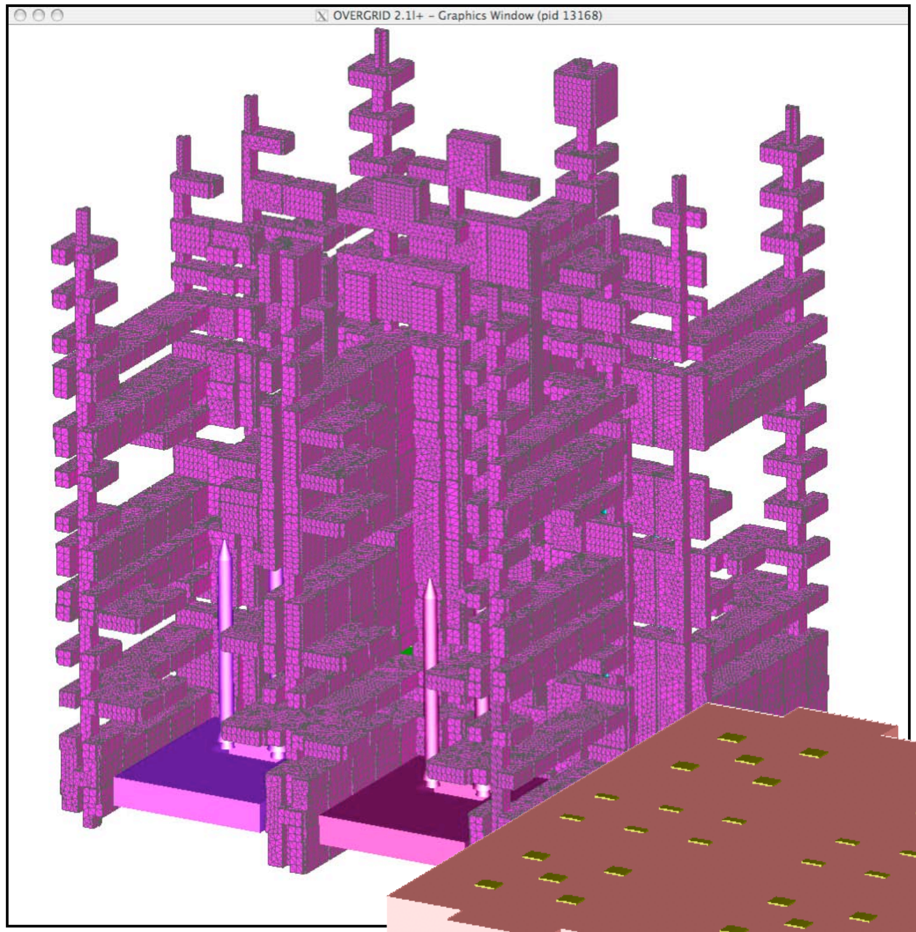
Cart3D Overview

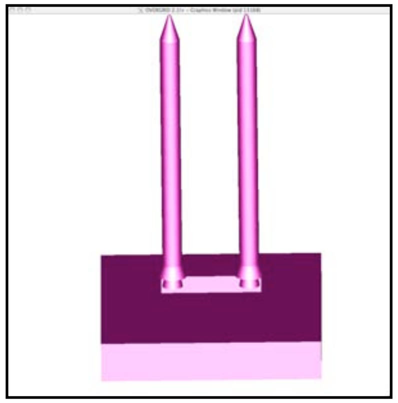
Automation and Scalability

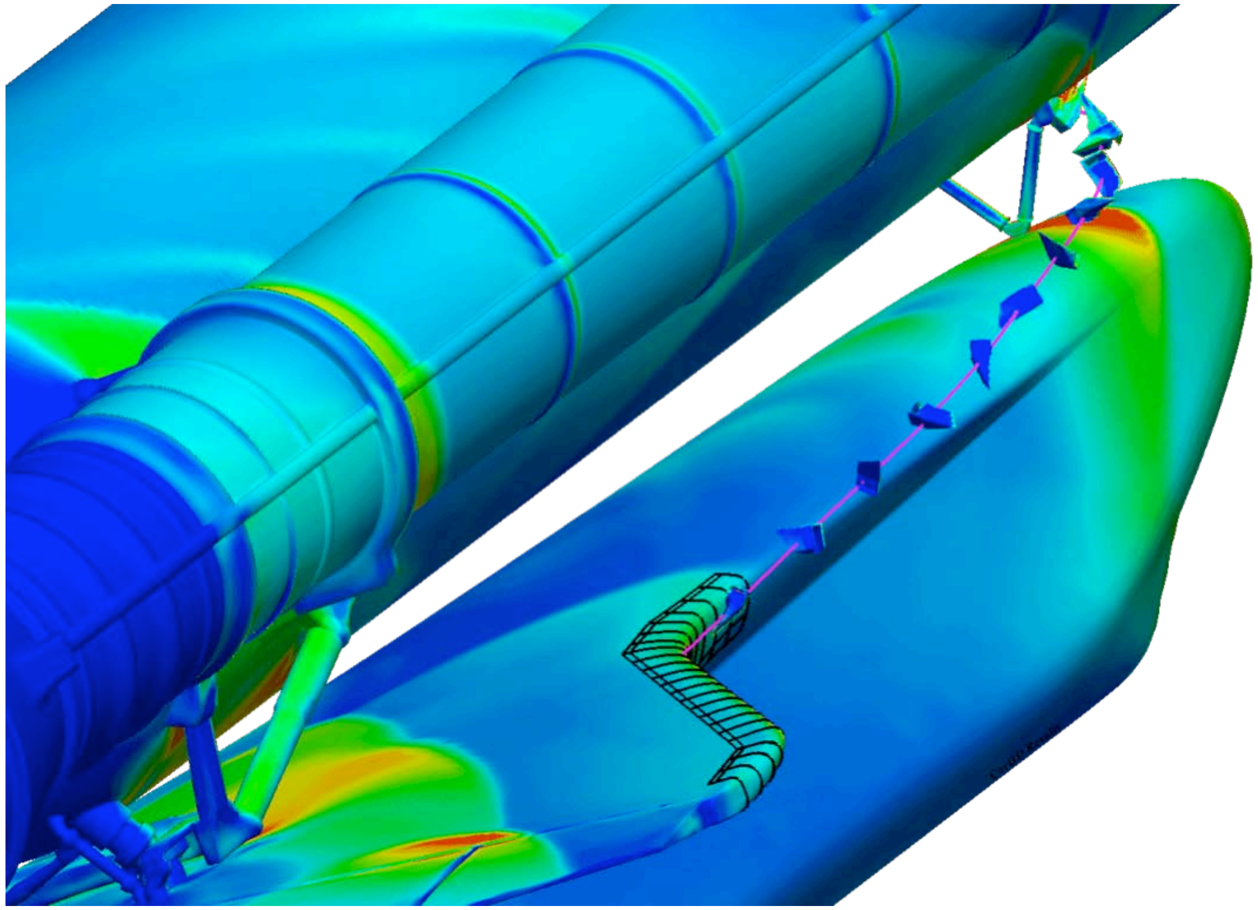
1. Surface geometry

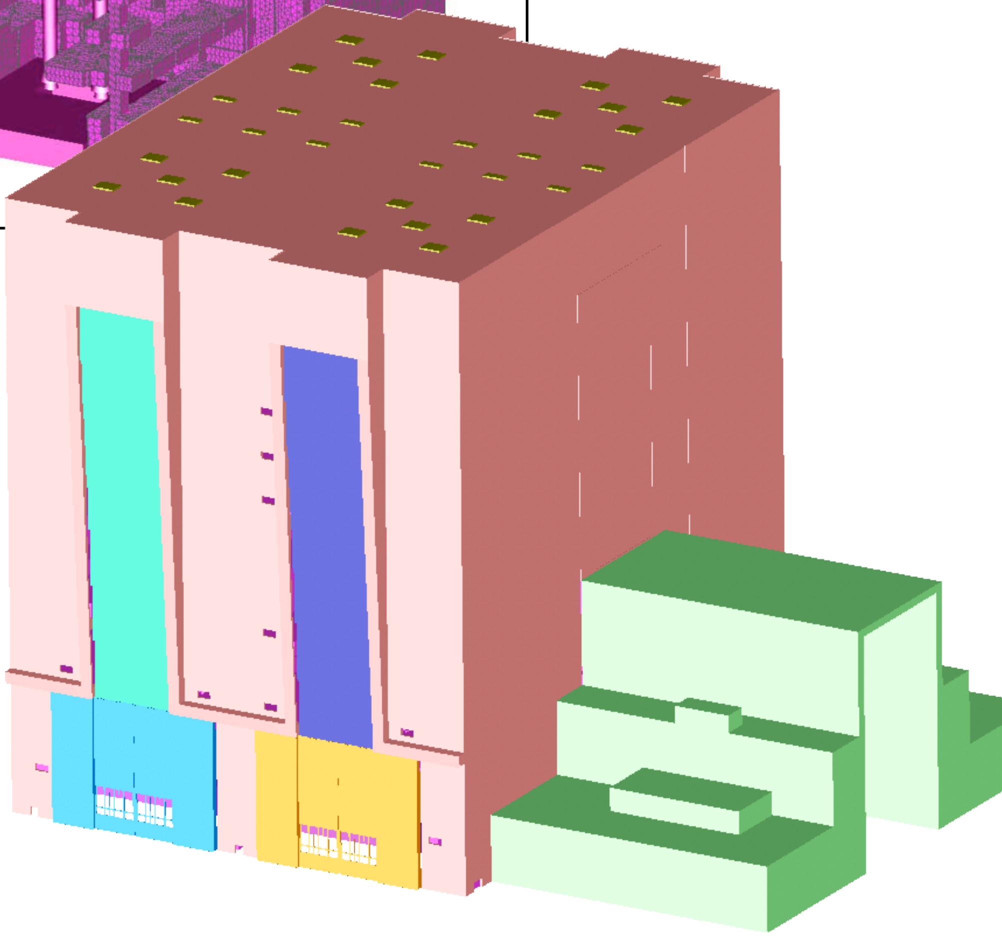
2. Volume mesh generator

3. Flow solver









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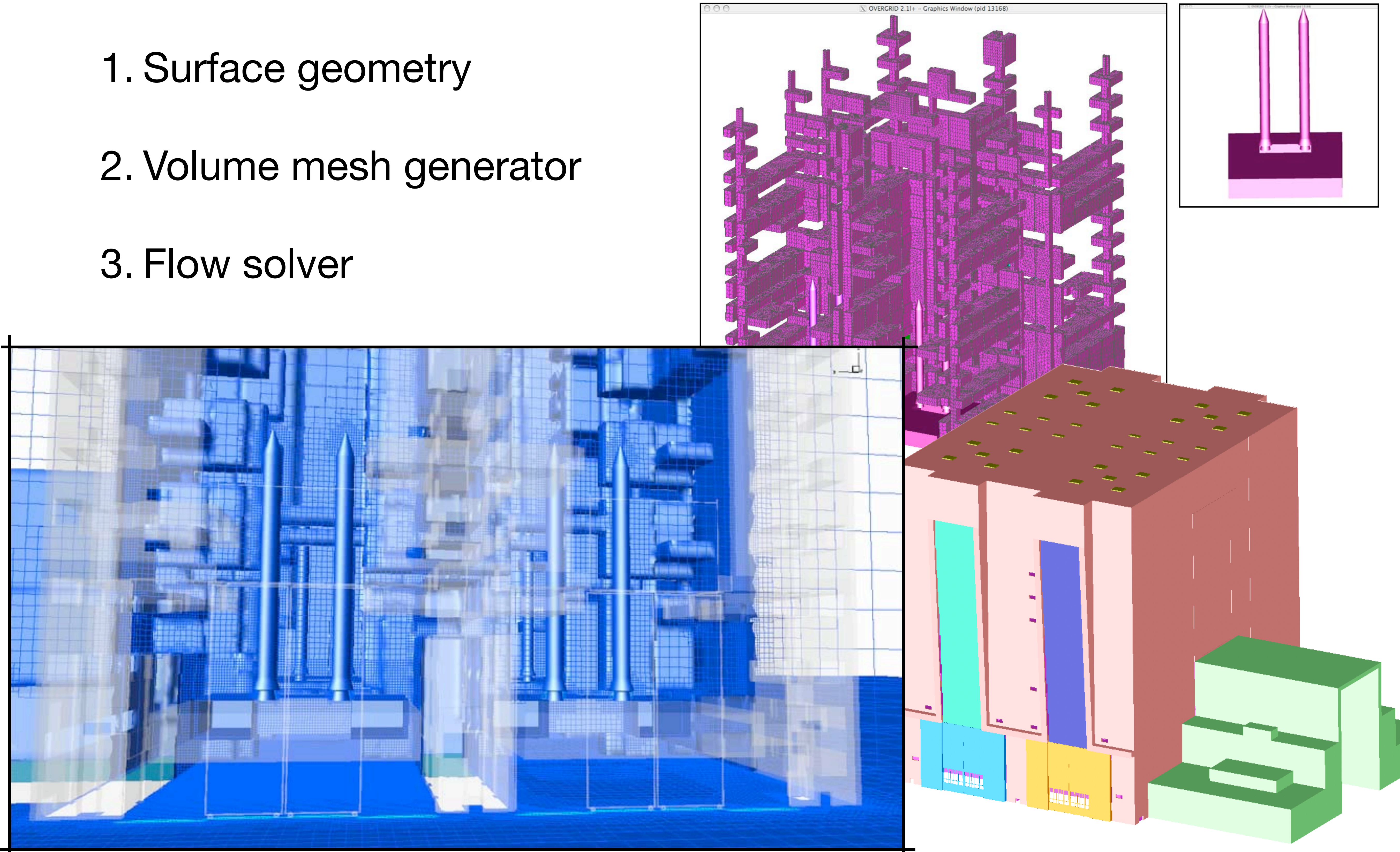
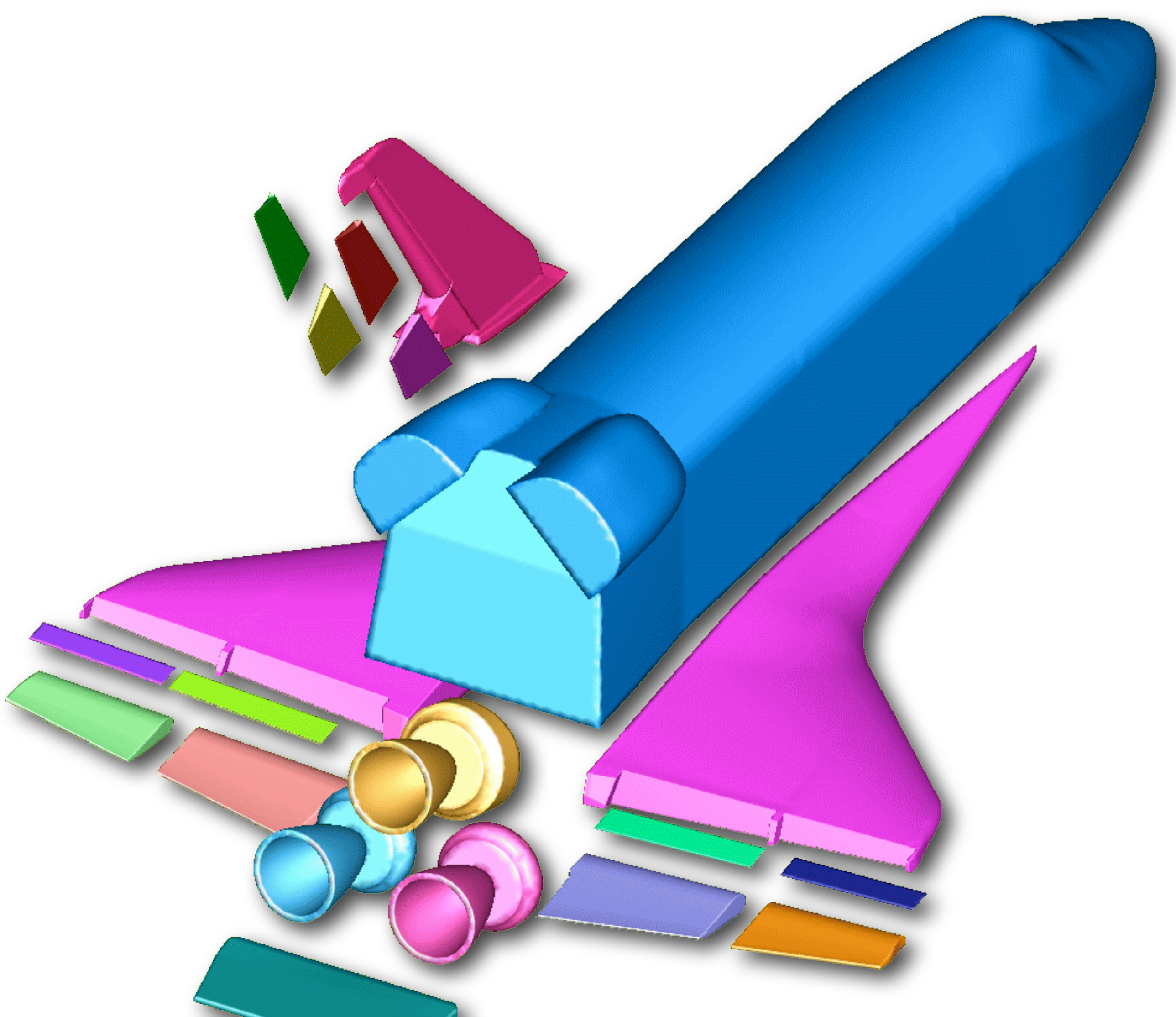
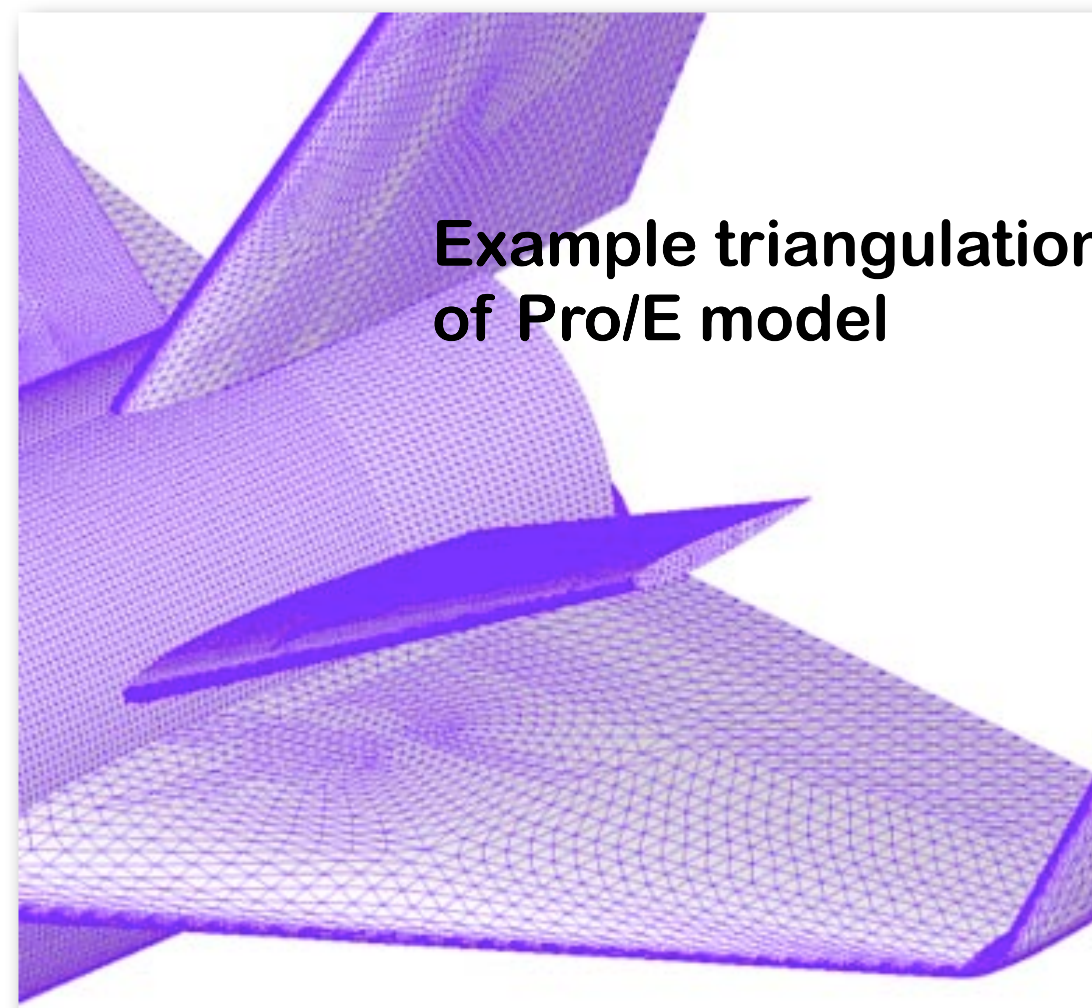


Cart3D

Surface Geometry

- Each solid (part) is an independent component
- "Water-tight" triangulation of individual components
  - Direct interface to native CAD parts and assemblies via CAPRI
- Intersect components to define a wetted-surface



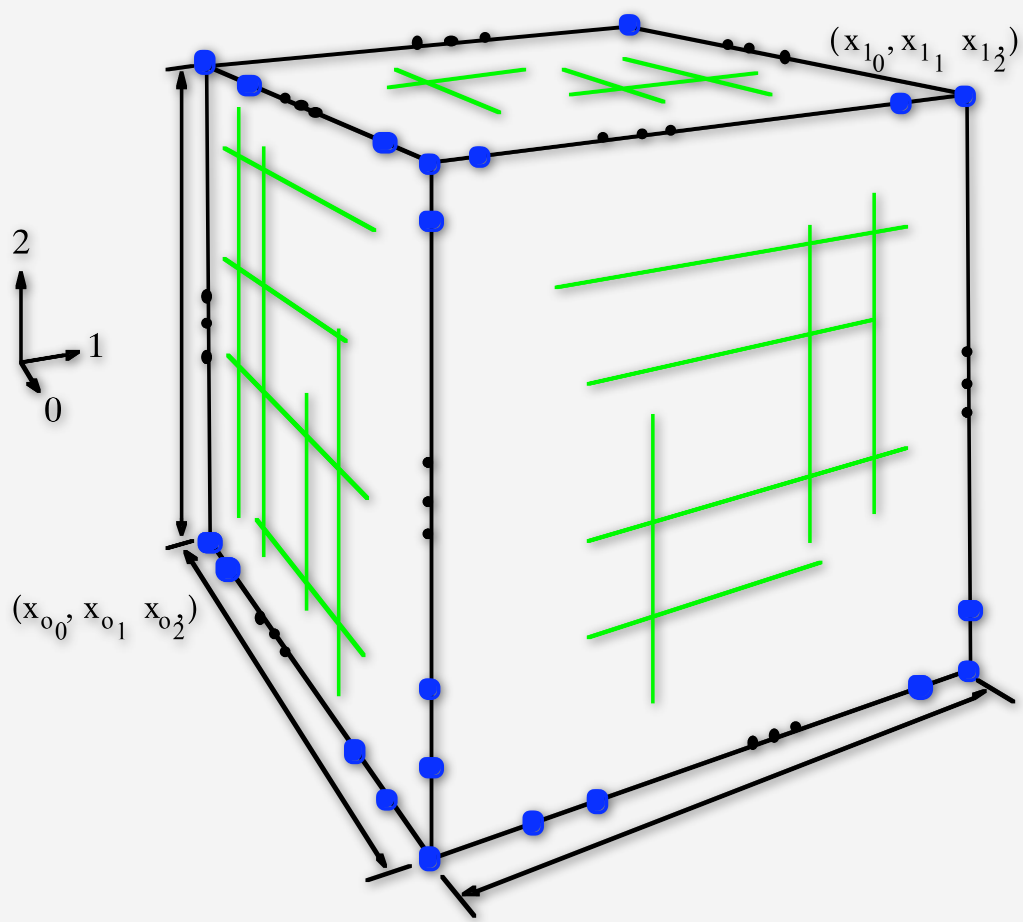


Example triangulation of Pro/E model

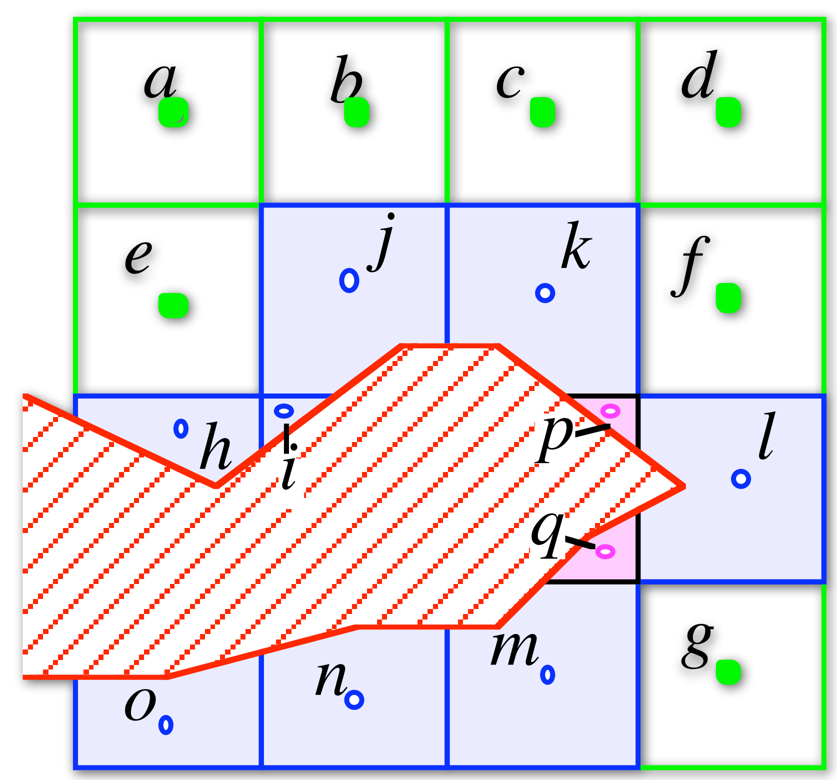
Cart3D

Non-Body-Fitted Cartesian Meshes

Basic Concepts



- All possible meshes fully specified
- Fast and robust



- Only two kinds of cells:
  - Very simple or very complex

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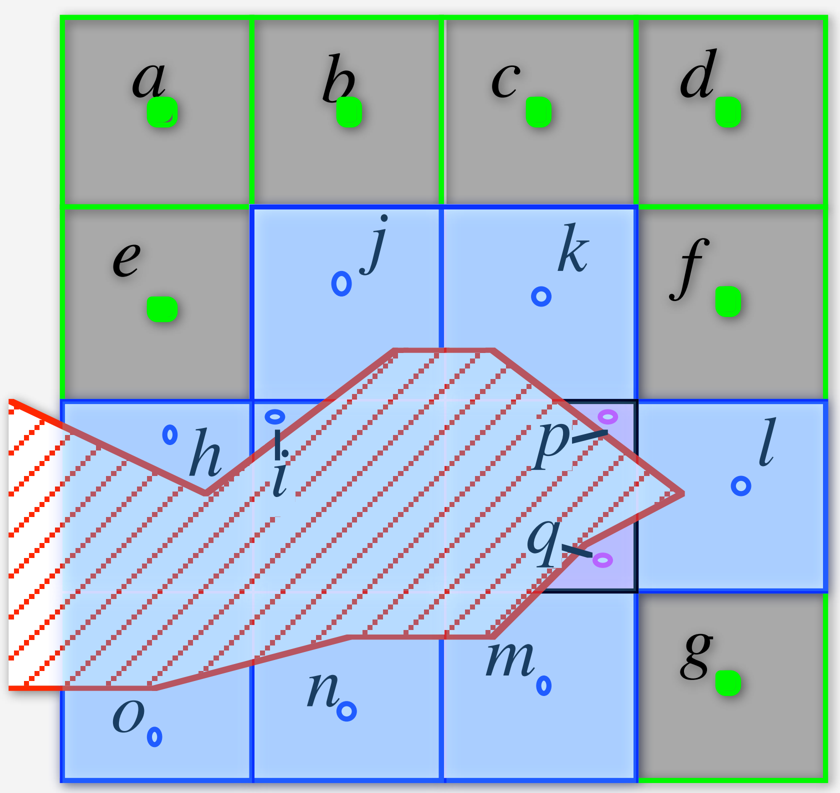
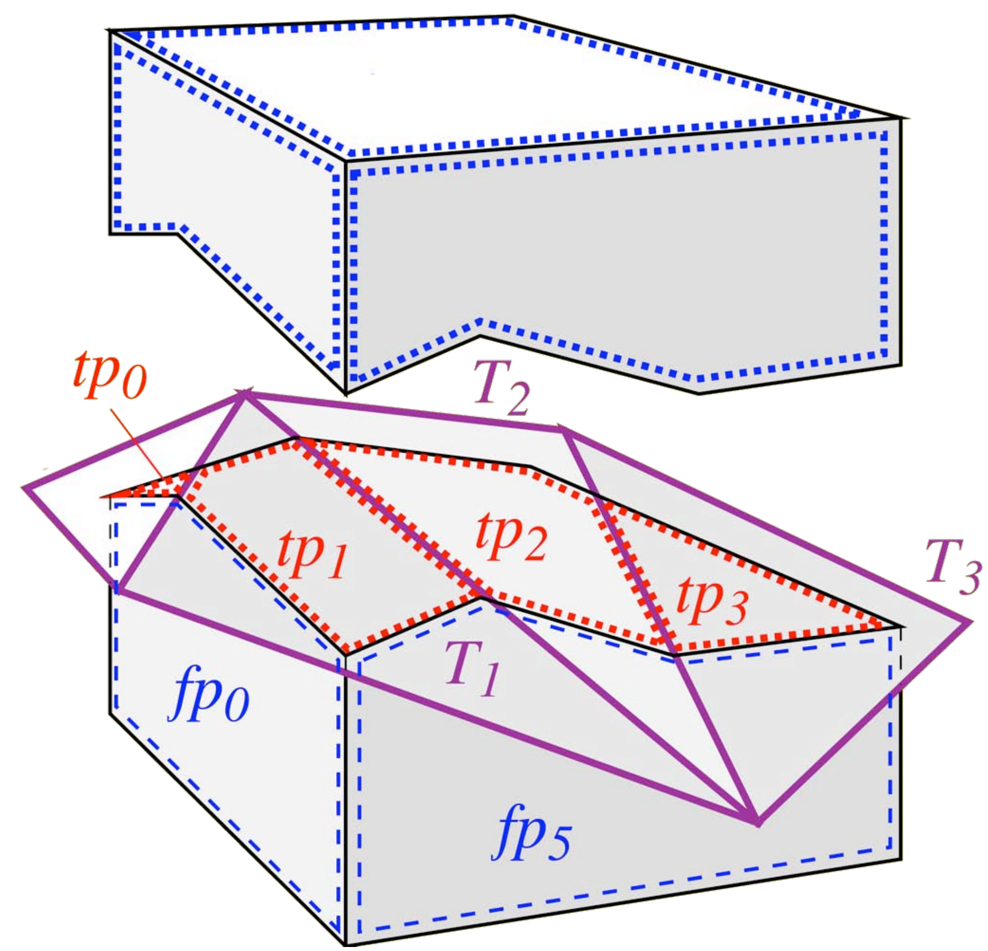
Non-Body-Fitted Cartesian Meshes

Basic Cell Types



- $\mathcal{O}(N^3)$ cells are regular hexahedra
- $\mathcal{O}(N^2)$ Cut-cells are general polyhedra

Cut-cell construction



1. Pierce points & triangle polygons
2. Cartesian face areas and centroids
3. Wall normals, cell volumes & centroids

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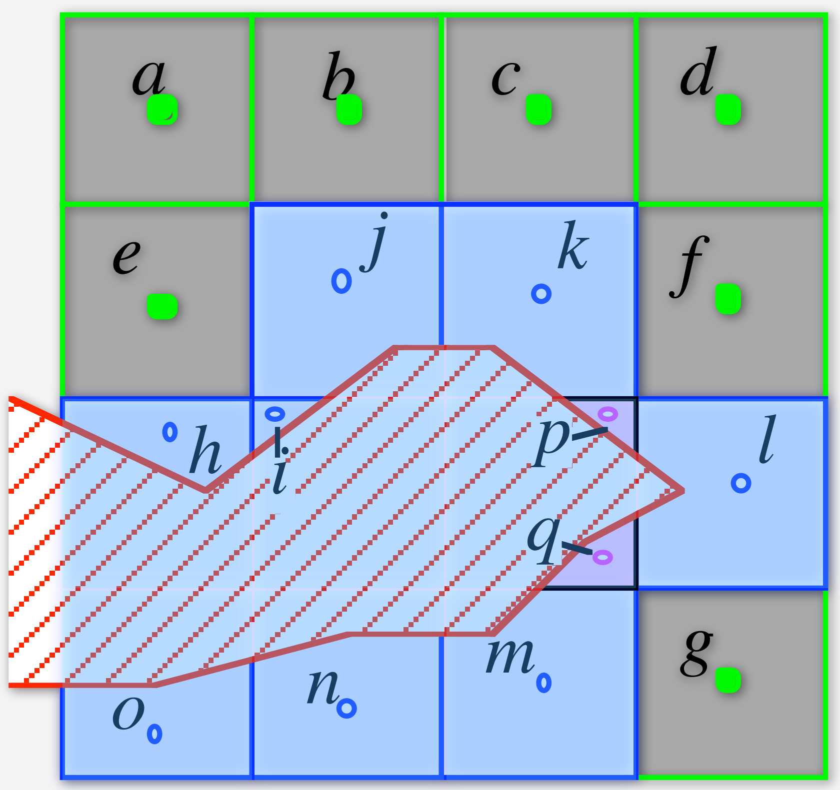
Non-Body-Fitted Cartesian Meshes

Basic Cell Types



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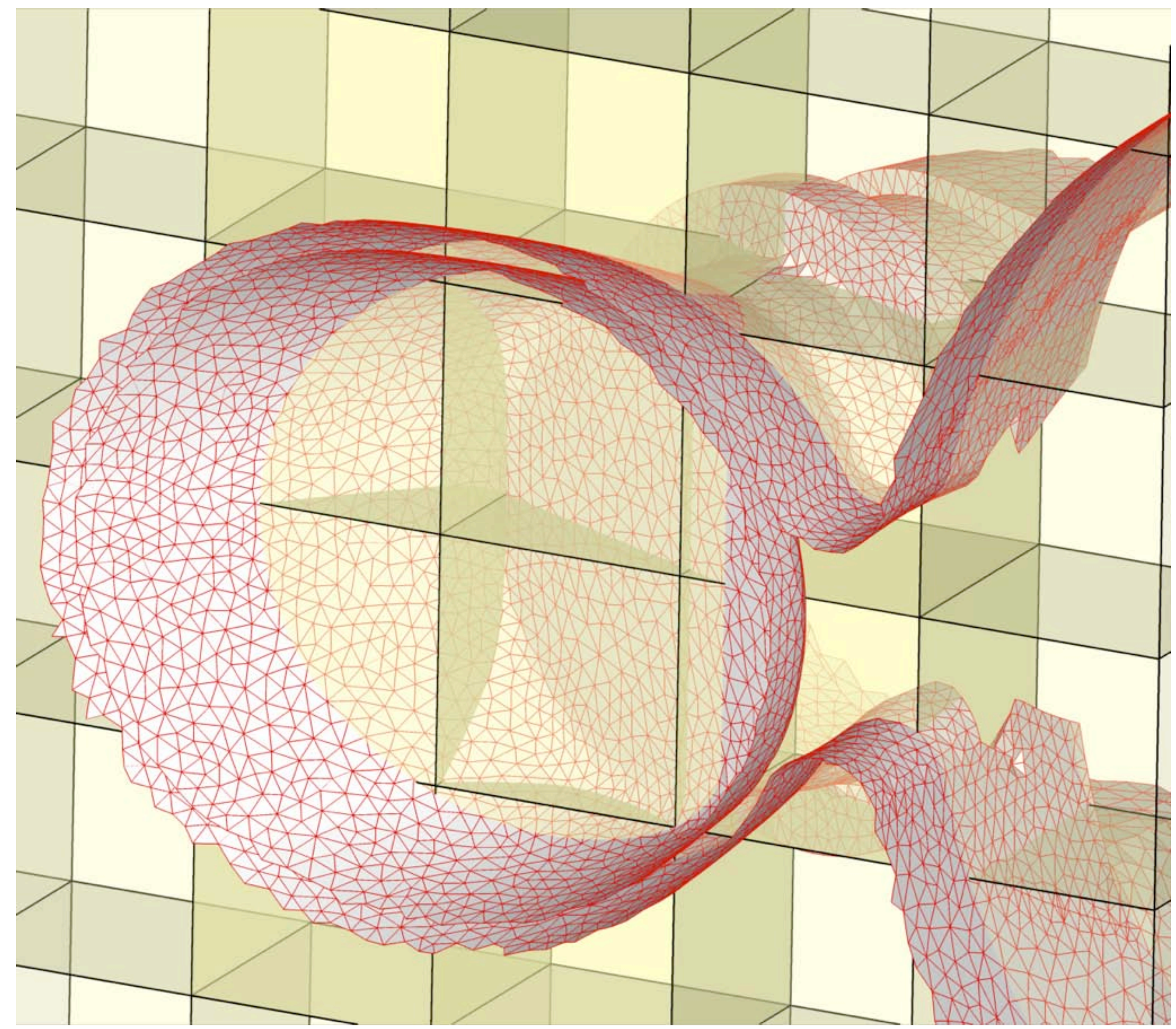
Cut-cells in practice

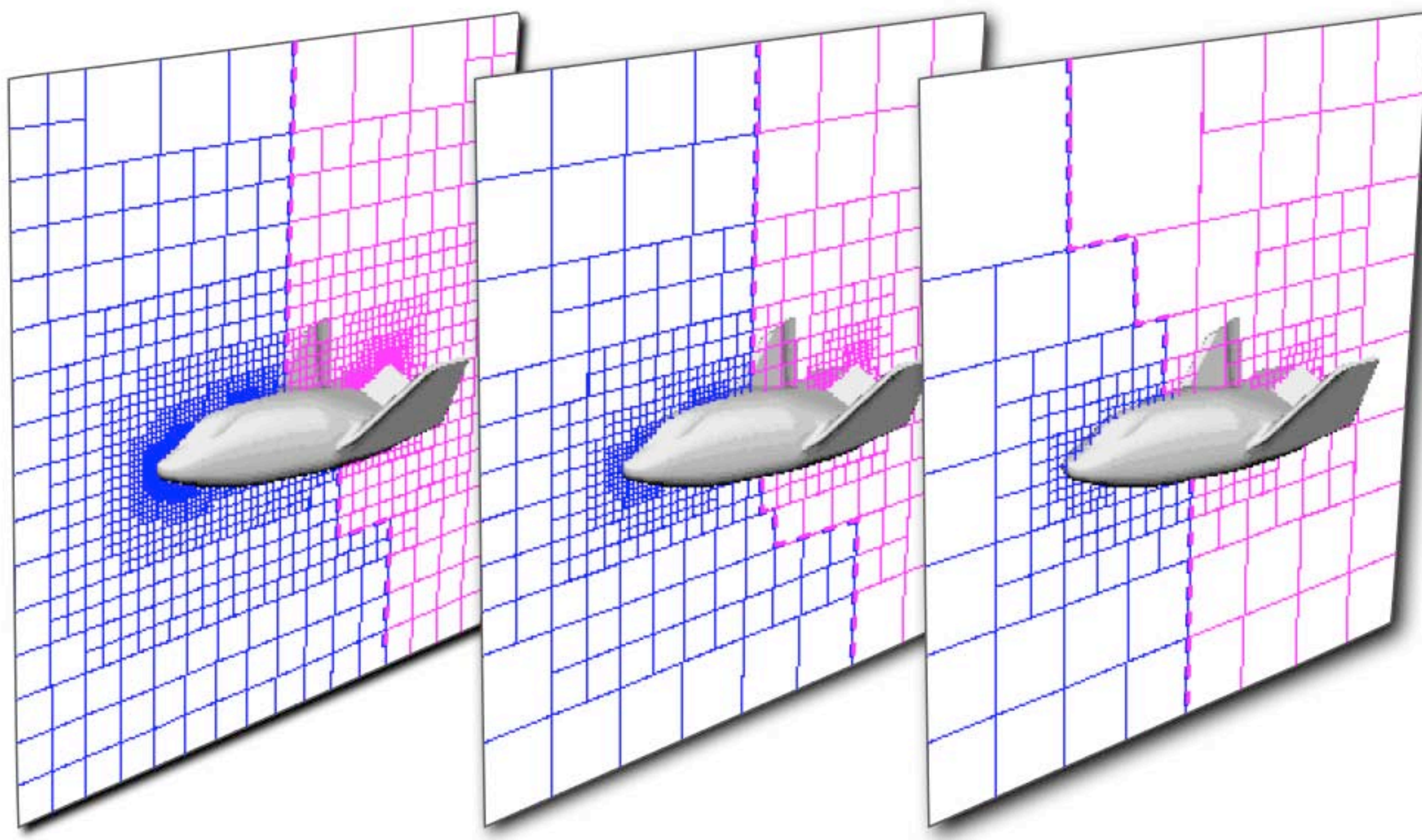


- Robust computational geometry algorithms allow configurations of arbitrary complexity

Cart3D

Steady-State Inviscid Flow Solver

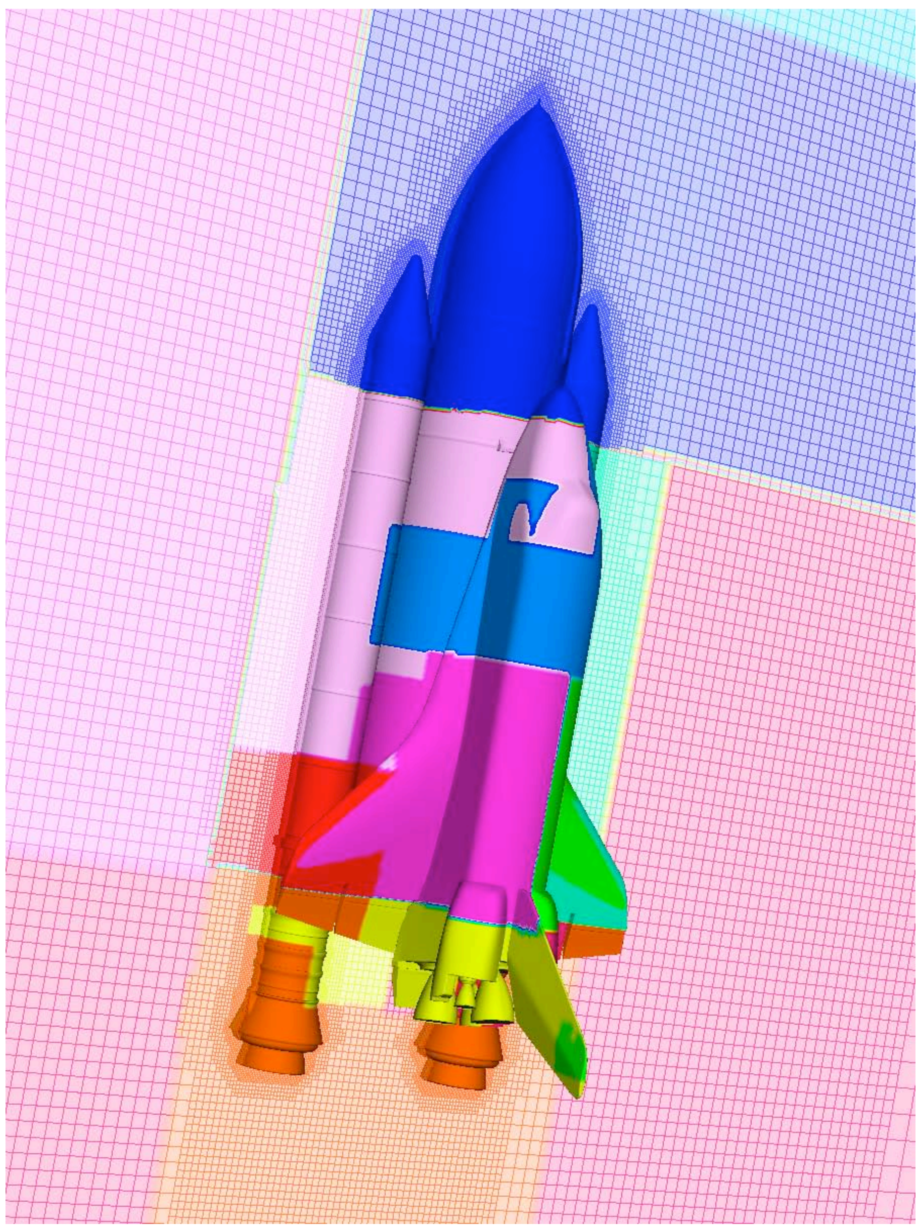
- Second-order accurate spatial discretization; van Leer flux vector splitting
- Runge-Kutta time marching with multigrid acceleration
- Domain decomposition and multigrid coarsening via space-filling curves

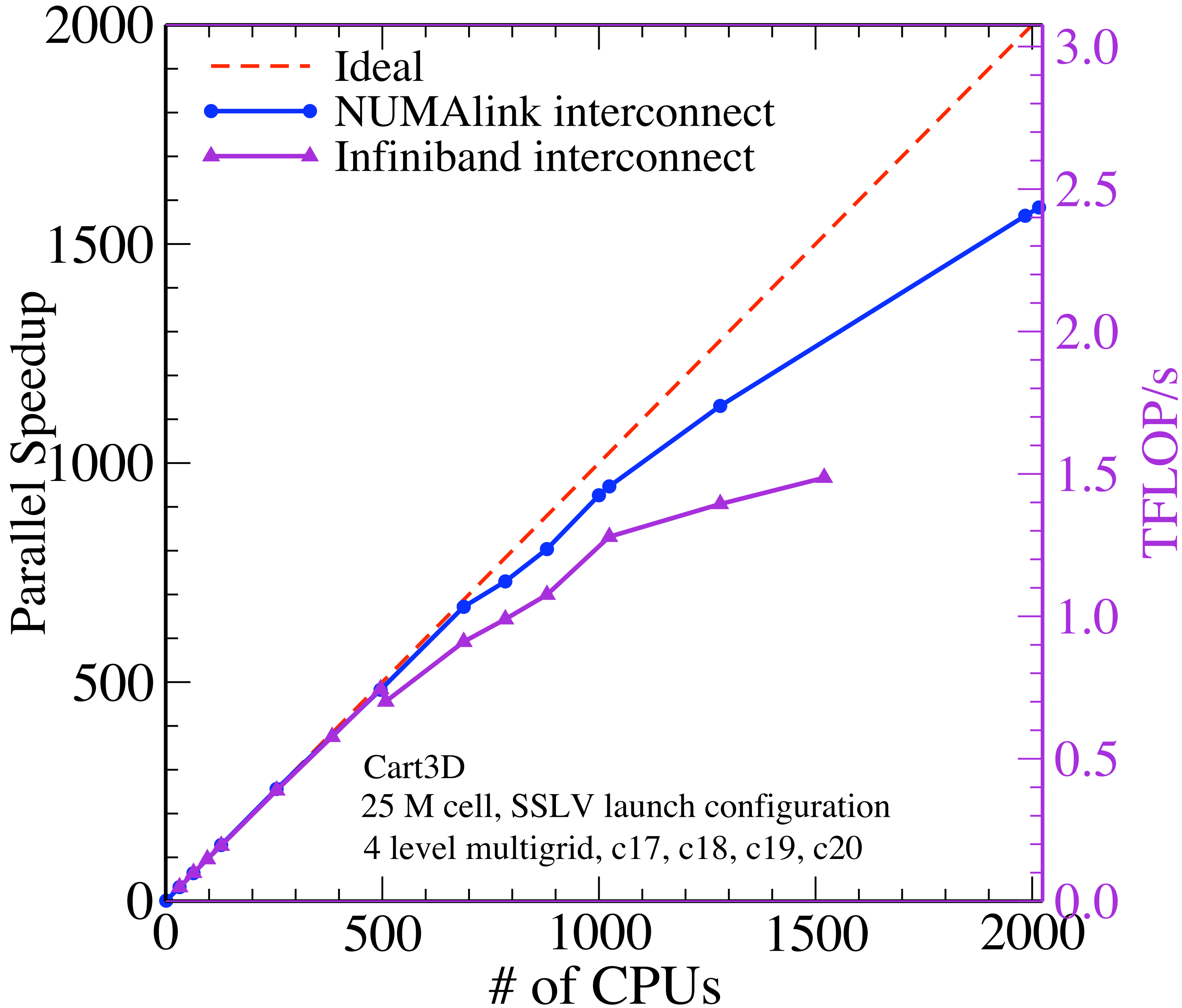


Cart3D

Scalability

- Example computation on NASA's Columbia supercluster

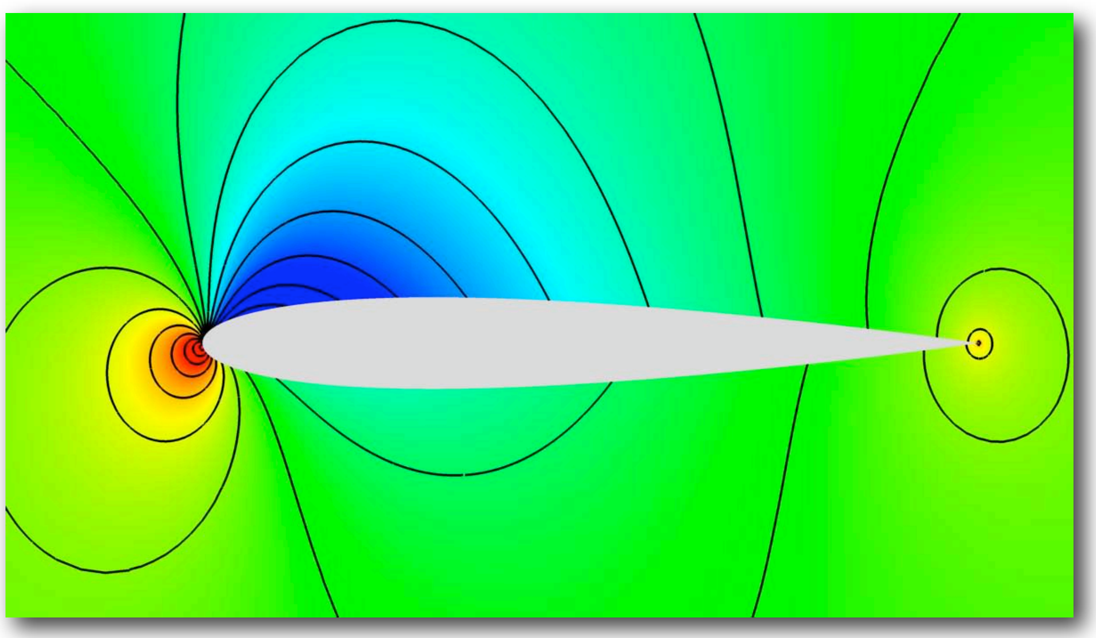


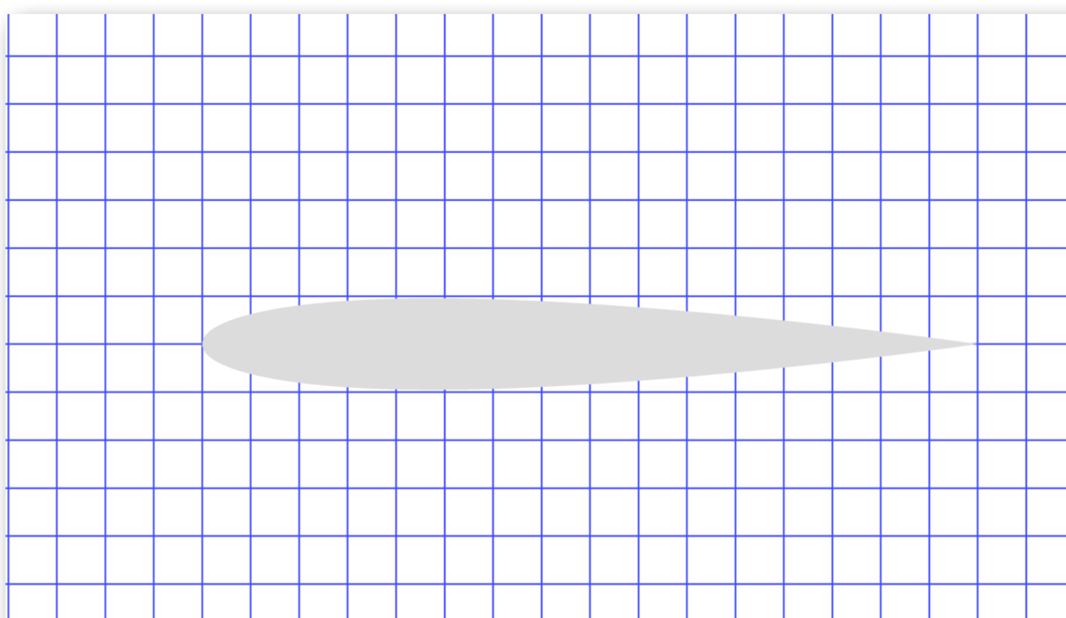


Error Estimation and Mesh Adaptation

Control of Numerical Errors in Flow Simulations

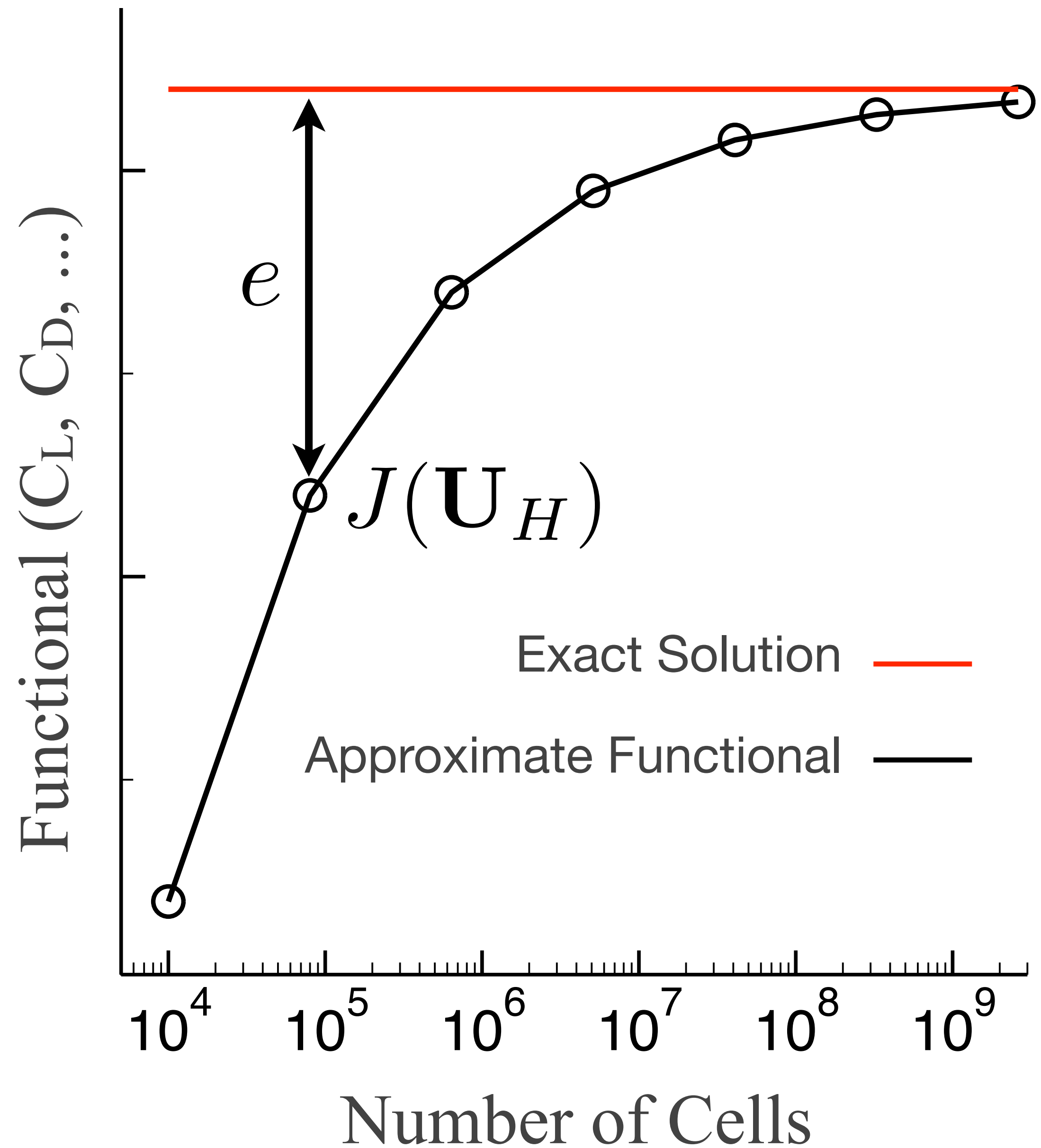
- Consider airfoil in subsonic inviscid flow. Our goal is to compute lift to a specified tolerance
- Where should the mesh be refined? How much?





Remember: our focus is on numerical errors (discretization errors) - the issue of modeling errors, i.e., how well do the solutions approximate experimental and flight data, is not addressed directly

Numerical Error
Uniform Mesh Refinement



Exact Solution: $\mathcal{J}$

- Numerical solution on a mesh with cell-size *H* gives approximate functional:

$$J(\mathbf{U}_H)$$

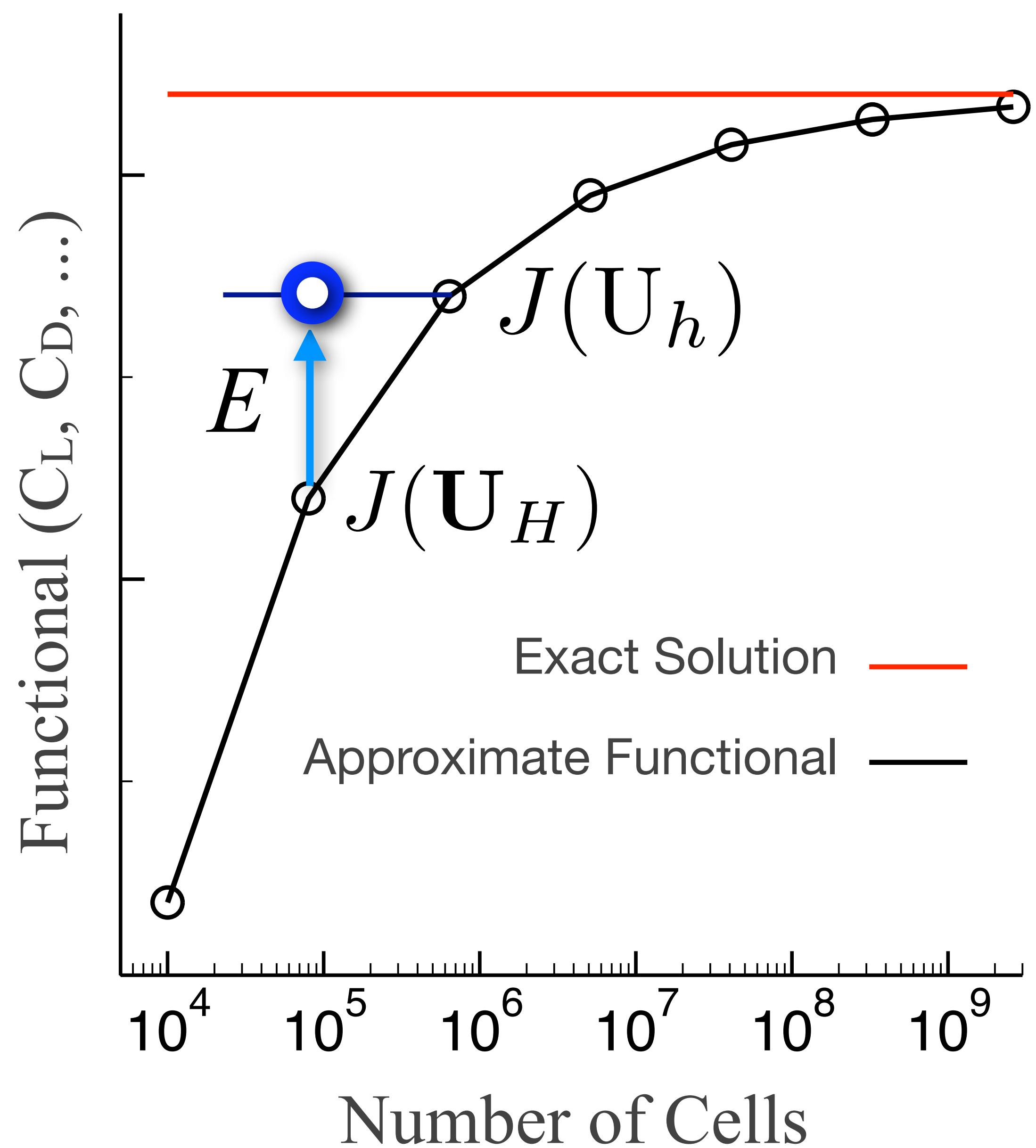
- Error in functional:

$$e = |\mathcal{J} - J(\mathbf{U}_H)|$$

- Goal is to estimate error as a function of the approximate flow solution:

$$e = f(\mathbf{U}_H)$$

Discrete Estimate of Numerical Error



- Consider simpler problem of computing relative error:

$$E = |J(\mathbf{U}_h) - J(\mathbf{U}_H)|$$

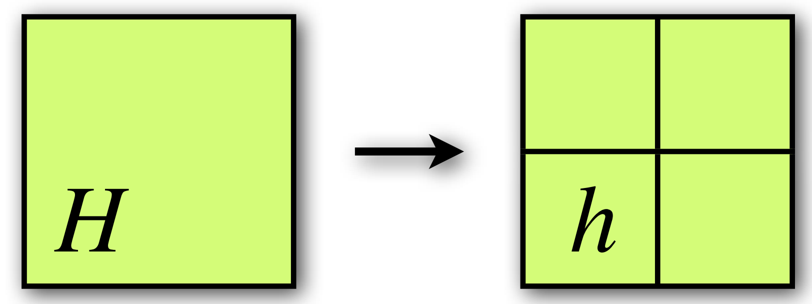
- We will use an adjoint solution on mesh *H* to estimate

$$E = f(\mathbf{U}_H, \psi_h)$$

Adjoint Error Estimates

- Consider a functional $J(\mathbf{U}_H)$ computed from the solution of Euler equations discretized on an affordable mesh with cell-size H:

$$R(\mathbf{U}_H) = 0$$



- In addition, consider an embedded mesh with cell-size h obtained via uniform refinement of the baseline mesh

- We seek to compute the error relative to the embedded mesh without solving the problem on the fine mesh

$$e_h = |J(\mathbf{U}_h) - J(\mathbf{U}_h^H)|$$

Venditti & Darmofal, 2002

- Estimate of functional on embedded mesh is obtained from Taylor series expansion of functional and residual equations about the coarse mesh solution

$$J(\mathbf{U}_h) \approx J(\mathbf{U}_h^H) + \frac{\partial J(\mathbf{U}_h^H)}{\partial \mathbf{U}_h}(\mathbf{U}_h - \mathbf{U}_h^H)$$

$$\mathbf{R}(\mathbf{U}_h) = 0 \approx \mathbf{R}(\mathbf{U}_h^H) + \frac{\partial \mathbf{R}(\mathbf{U}_h^H)}{\partial \mathbf{U}_h}(\mathbf{U}_h - \mathbf{U}_h^H)$$

- These equations are combined to give

$$J(\mathbf{U}_h) \approx J(\mathbf{U}_h^H) - \psi_h^{\mathrm{T}}\,\mathbf{R}(\mathbf{U}_h^H)$$

where ψ satisfies the adjoint equation

$$\left[\frac{\partial \mathbf{R}(\mathbf{U}_h^H)}{\partial \mathbf{U}_h}\right]^{\mathrm{T}} \psi_h = \frac{\partial J(\mathbf{U}_h^H)}{\partial \mathbf{U}_h}^{\mathrm{T}}$$

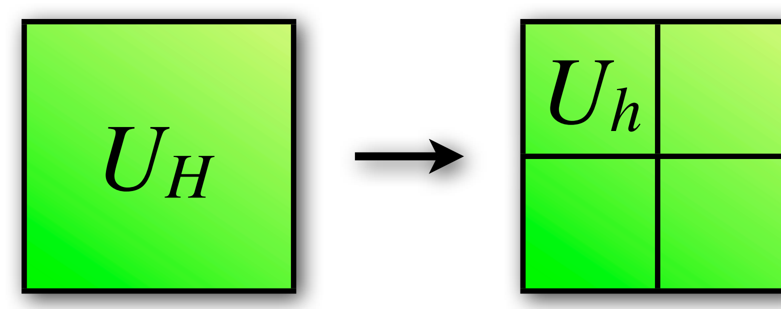
Adjoint Correction and Error Bound

- Since the adjoint solution is not known on the embedded mesh, we use an approximate solution from the coarse mesh to obtain

$$J(\mathbf{U}_h) \approx J(\mathbf{U}_h^H) - (\psi_h^H)^{\mathrm{T}}\mathbf{R}(\mathbf{U}_h^H) - (\psi_h - \psi_h^H)^{\mathrm{T}}\mathbf{R}(\mathbf{U}_h^H)$$

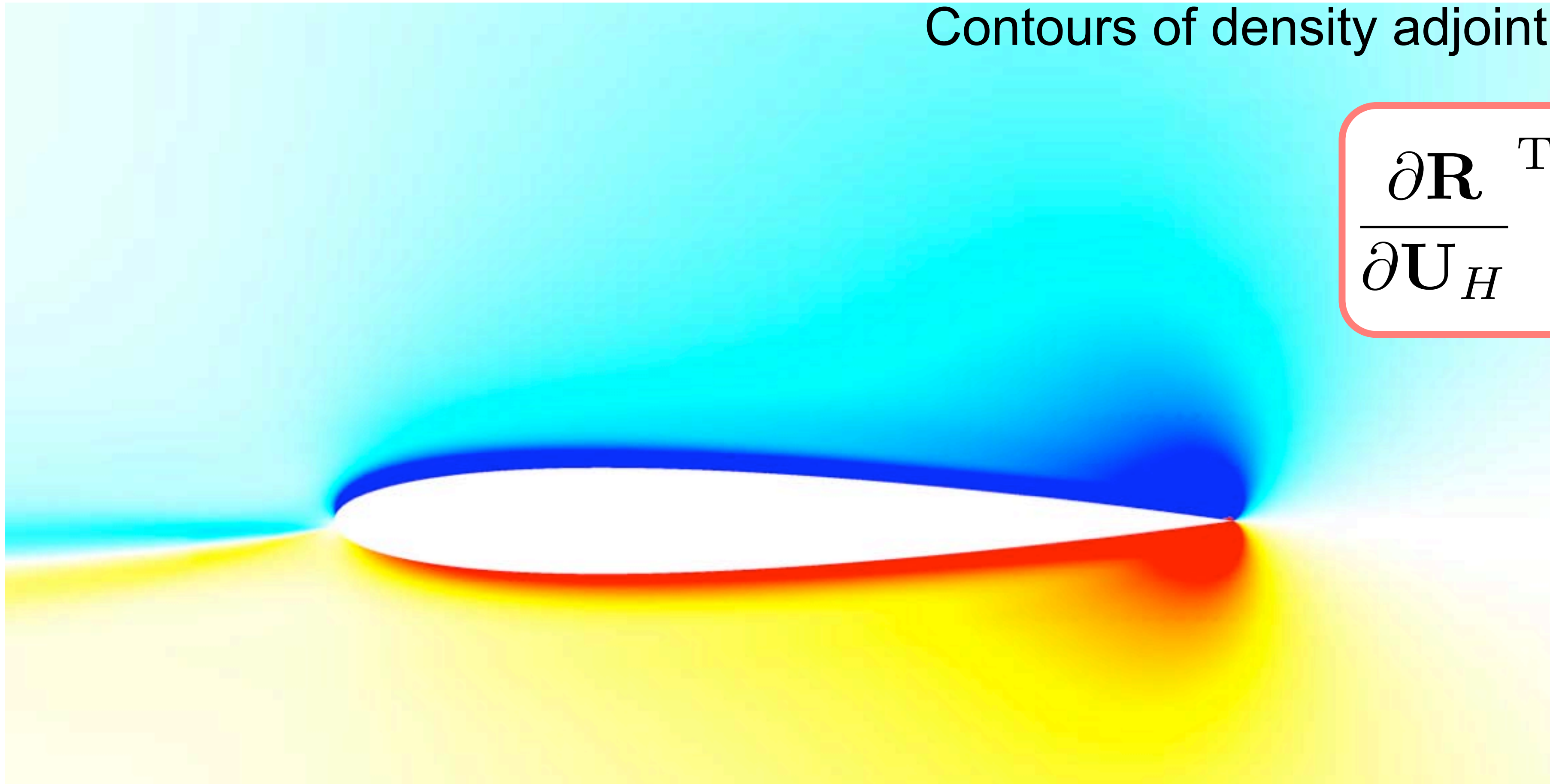
Adjoint Correction   Remaining Error

- Use piecewise quadratic (Q), linear (L) and constant (C) reconstruction operators to lift solutions from coarse mesh to embedded mesh



- How do we interpret adjoint solutions? And how do we use the adjoint correction and remaining error terms?

Example Adjoint Solution for Lift



$$\frac{\partial \mathbf{R}}{\partial \mathbf{U}_H}^{\mathrm{T}} \psi_H = \frac{\partial J}{\partial \mathbf{U}_H}^{\mathrm{T}}$$

- Adjoints identify flow regions important for lift: trailing-edge singularity, near-body region and leading-edge stagnation line
- Control problem
  - Optimal shape design: adjust design variables to control the flow and improve performance
  - Error analysis: adjust mesh refinement to control discretization errors

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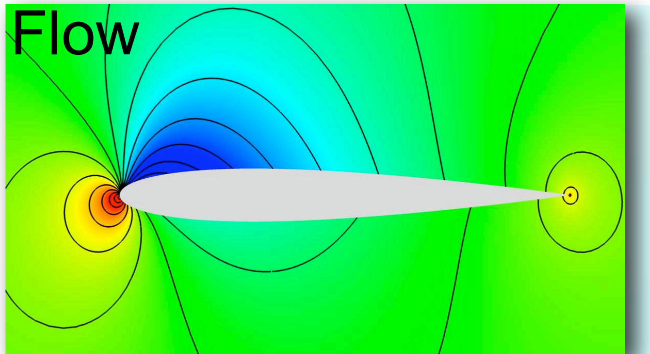

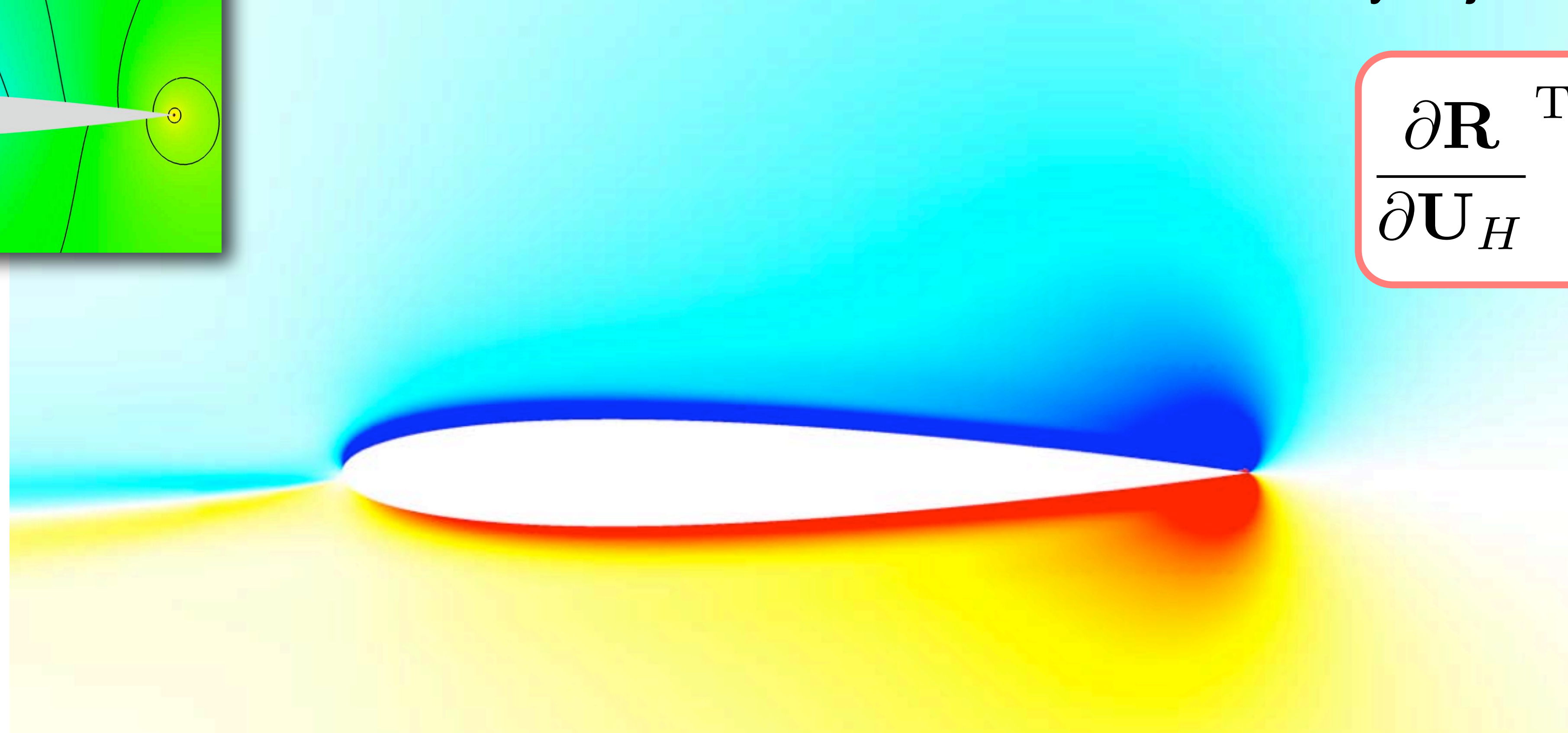

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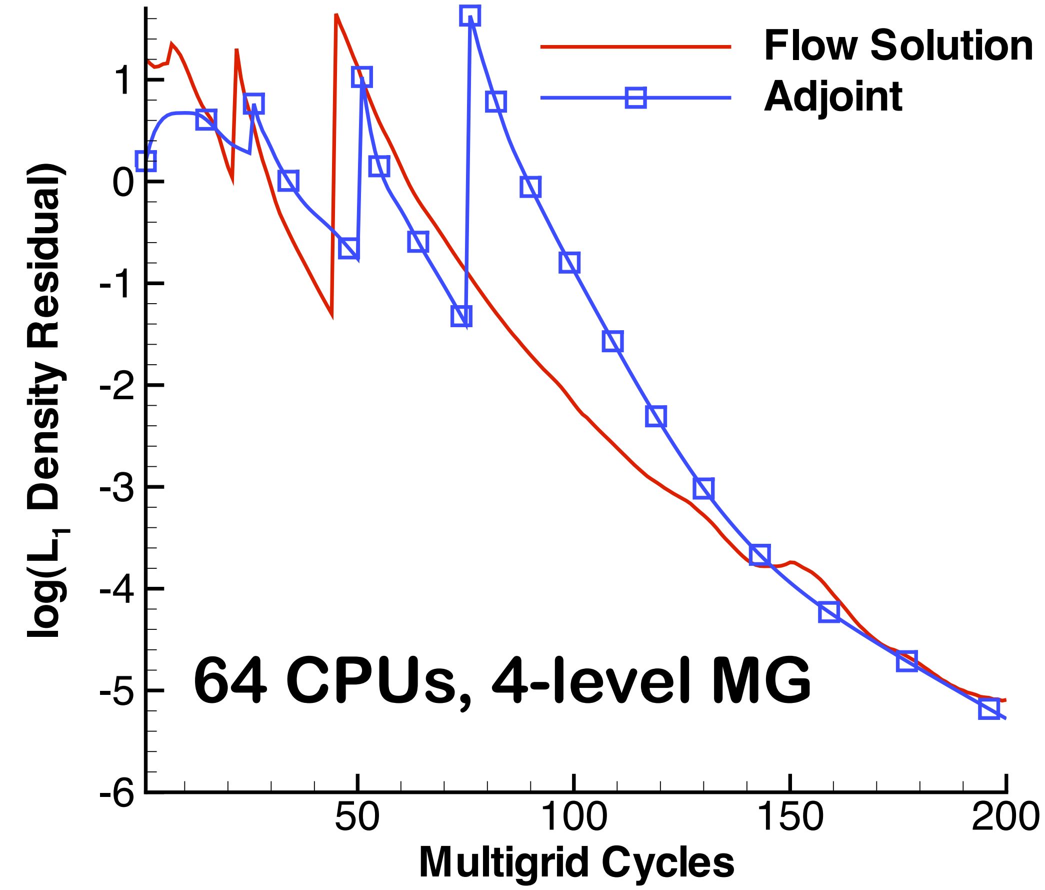
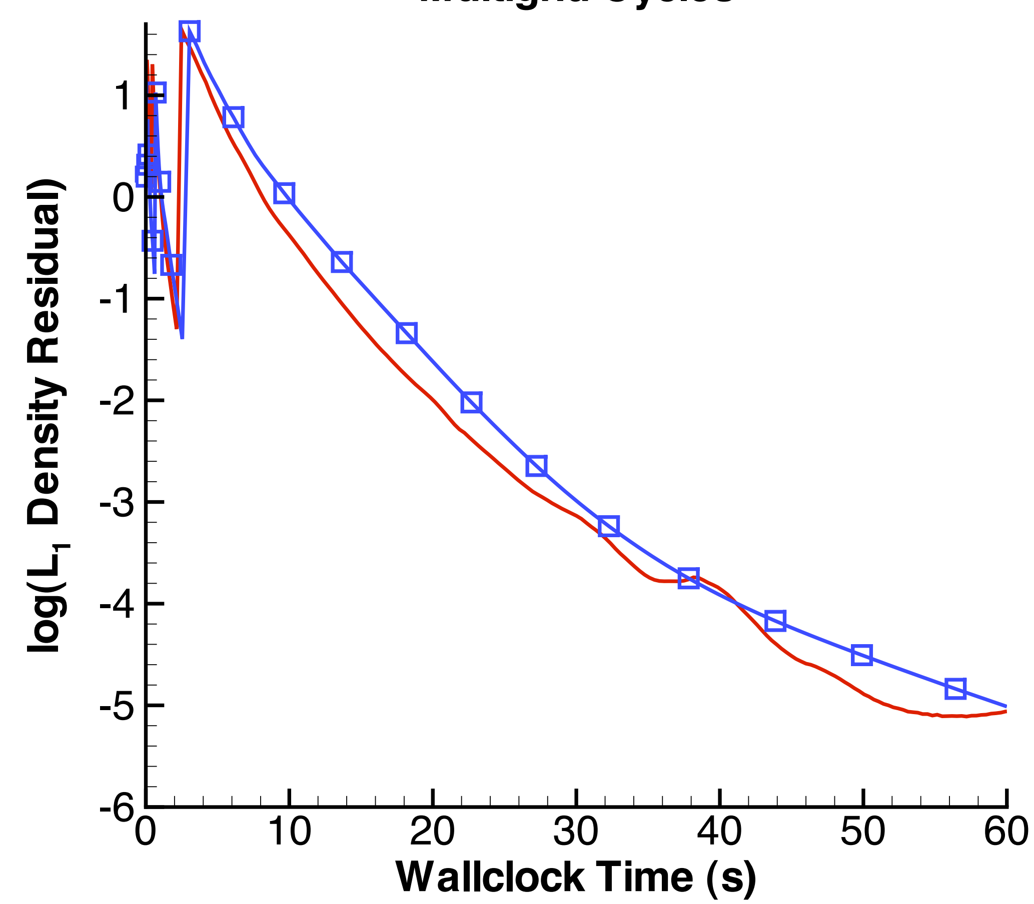
Adjoint Implementation

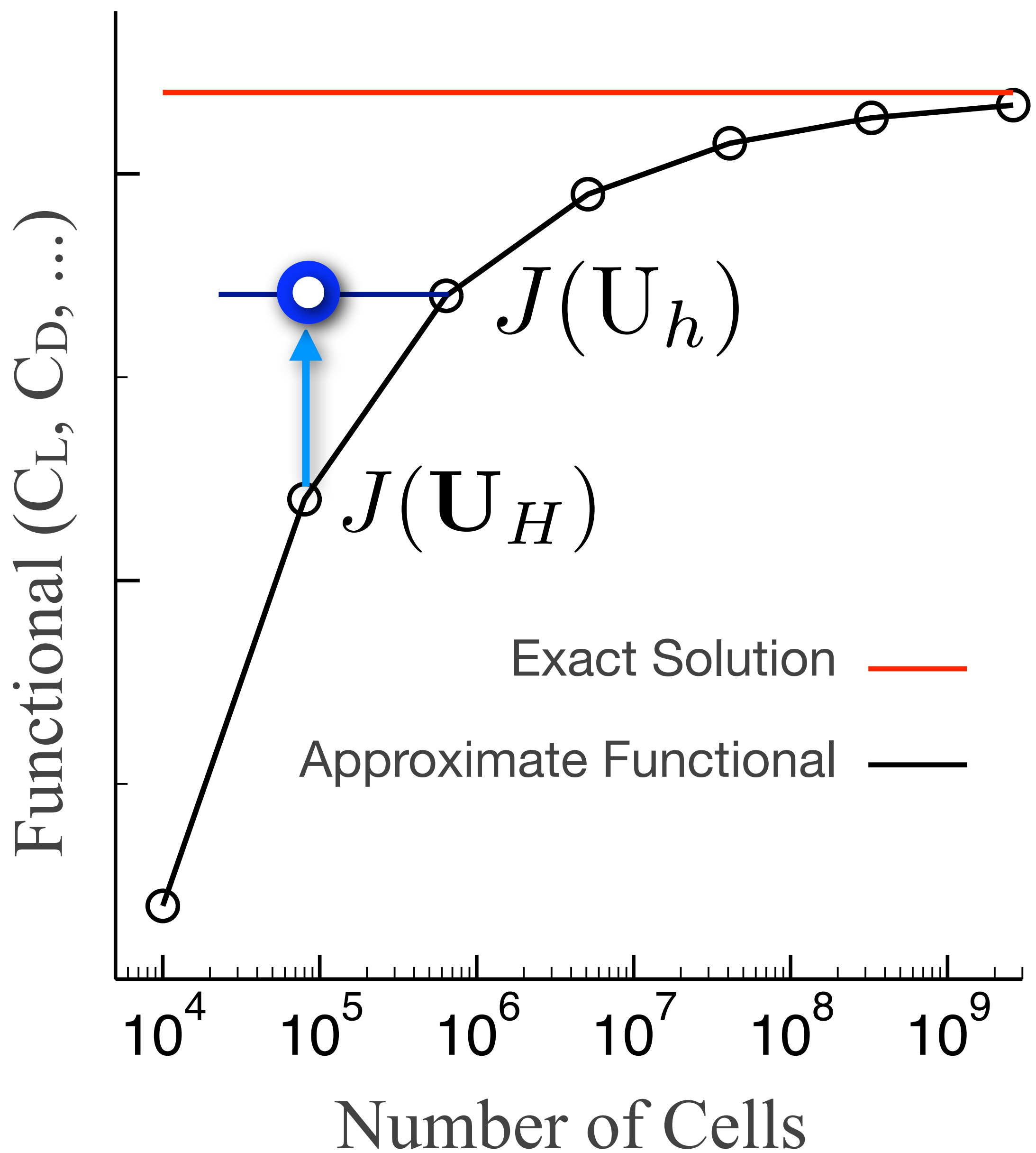
- Exact linearization of flow solver
  - Except for limiters (assumed constant)
- Duality preserving approach
  - Adopt RK5 time marching with multigrid and domain decomposition schemes of the flow solver
  - Minor modifications in gradient updates in MG restriction to reduce wall-clock time
- In practice, convergence of adjoint multigrid is not as robust as flow solver
  - Positivity preserving prolongation operator from flow solver cannot be used directly in the adjoint solver



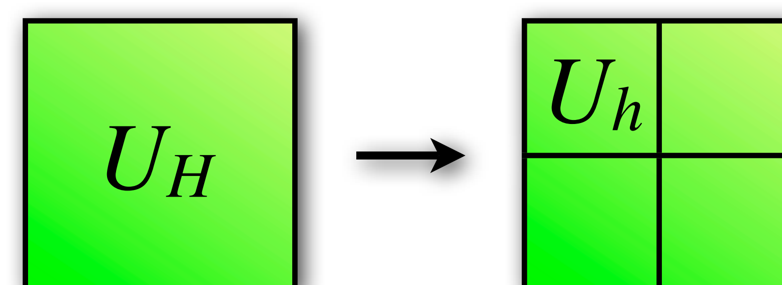


Adjoint Correction

$$\boxed{J(\mathbf{U}_h) \approx J(\mathbf{U}_h^H) - (\psi_h^H)^{\mathrm{T}} \mathbf{R}(\mathbf{U}_h^H)} - (\psi_h - \psi_h^H)^{\mathrm{T}} \mathbf{R}(\mathbf{U}_h^H)$$



- Predict functional on a fine mesh with cell-size *h* from a coarse mesh solution with cell size *H*



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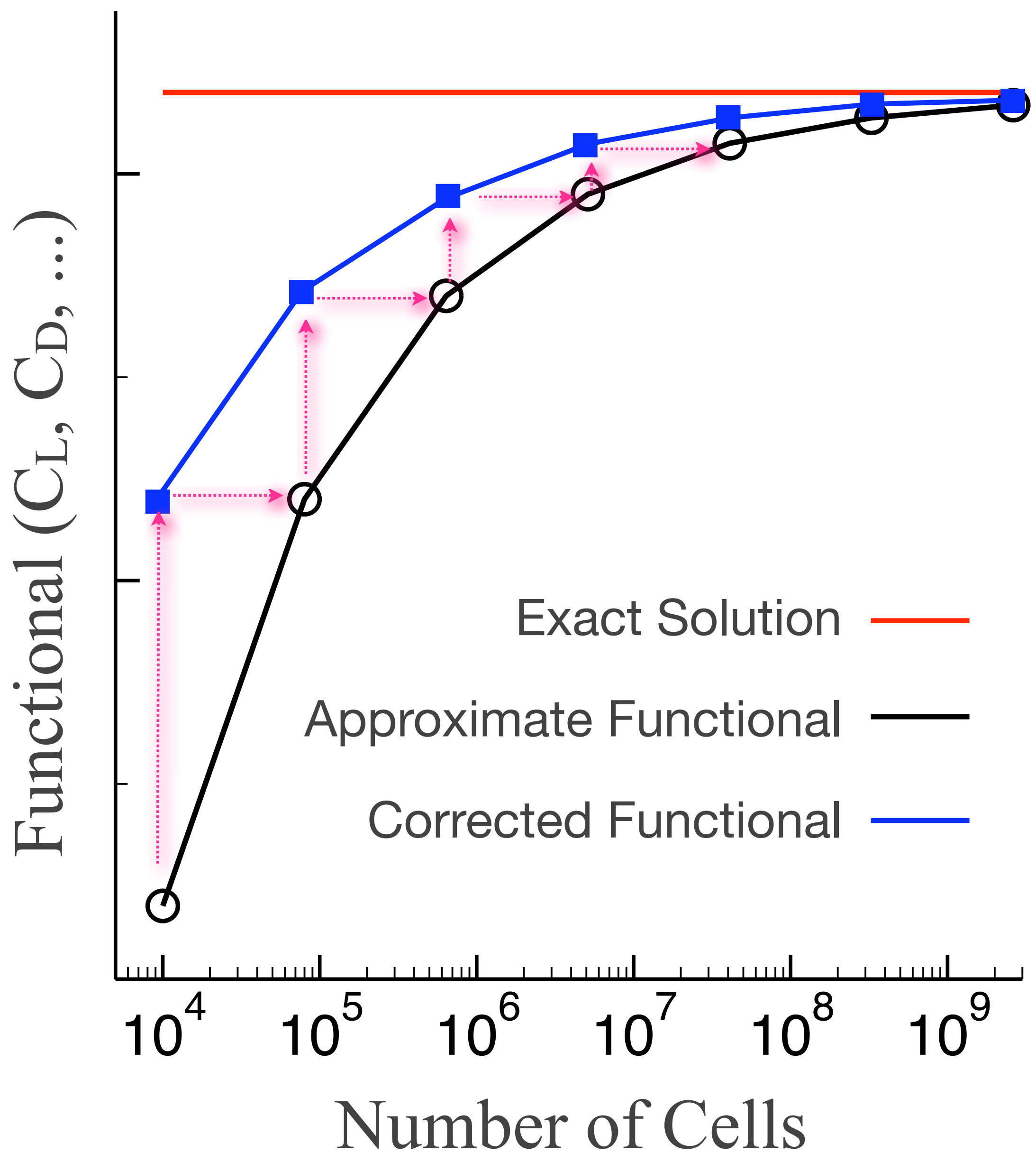

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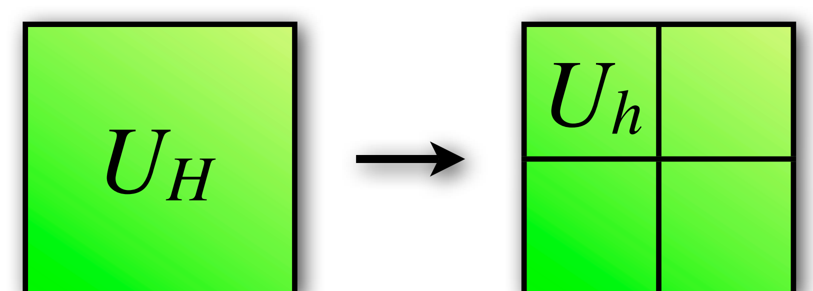
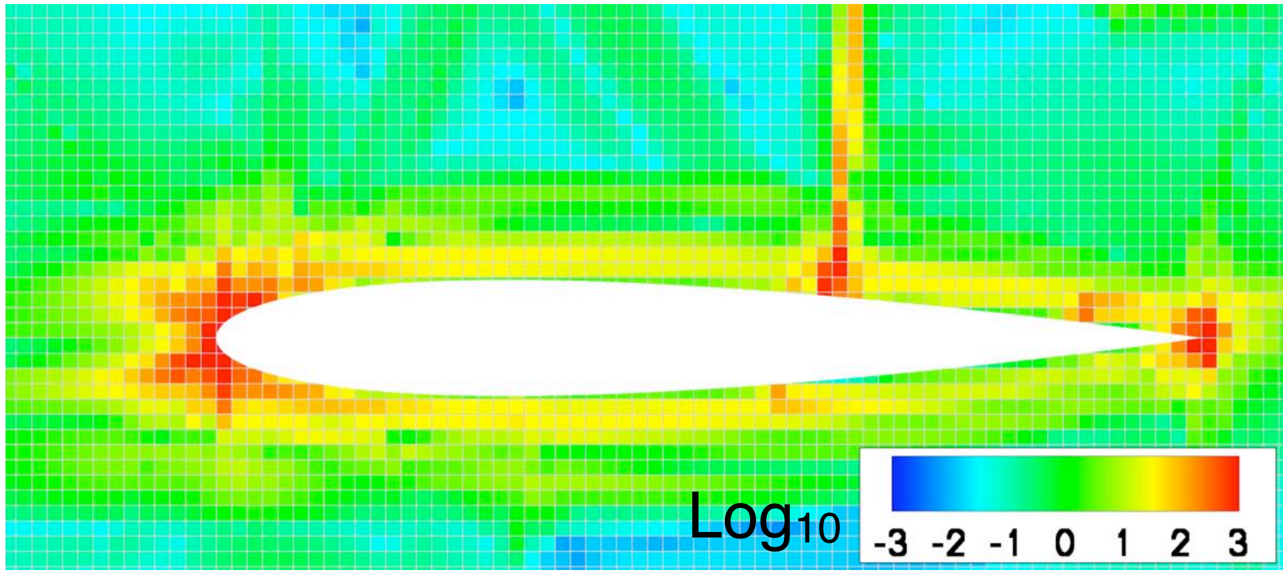


Error Bound Estimate

$$J(\mathbf{U}_h) \approx J(\mathbf{U}_h^H) - (\psi_h^H)^{\mathrm{T}}\mathbf{R}(\mathbf{U}_h^H) - \boxed{(\psi_h - \psi_h^H)^{\mathrm{T}}\mathbf{R}(\mathbf{U}_h^H)}$$



- Bound on remaining error in each coarse cell *k*

$$e_k = \sum \left| (\psi_{\mathrm{L}} - \psi_{\mathrm{C}})^{\mathrm{T}} \mathbf{R}(U_{\mathrm{L}}) \right|_k$$

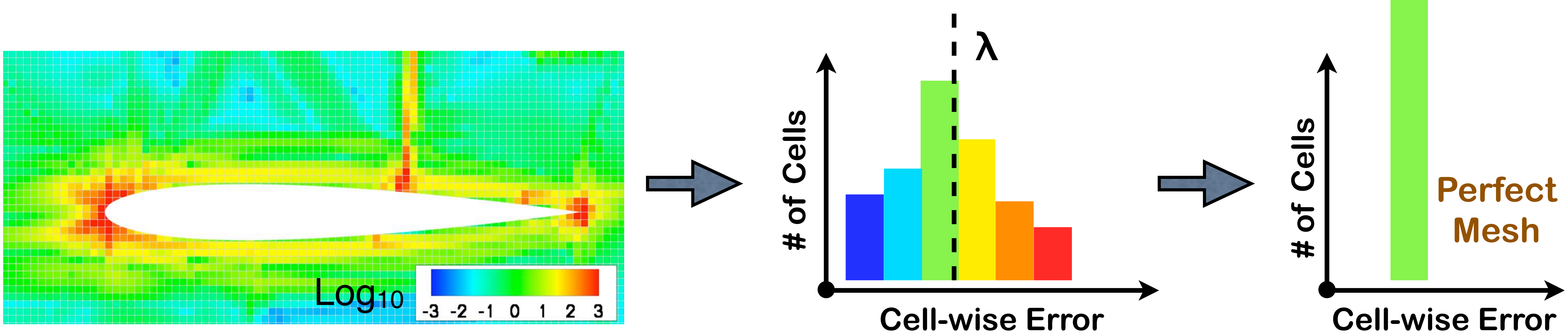
- Net functional error $E = \sum_{k=0}^{N} e_k$

- Given a user specified tolerance TOL, termination criterion is satisfied when E < TOL

Refinement Parameter

- Define maximum allowable error level in each coarse cell via equidistribution: t = TOL / N
- Refinement parameter in each cell is given by $r_k = \dfrac{e_k}{t}$
- Refine cells for which $r_k > \lambda$
  where $\lambda \geq 1$ is a global threshold factor

Error Histograms



How do you choose λ at each adaptation cycle to minimize simulation cost? ... see AIAA 2008-0725

Results
Focus on applications

Part A. Classic examples

1. Typical Launch Abort Vehicle database

2. Parametric studies of Launch Vehicles

3. Transport Aircraft

4. Quiet Supersonic Cruise

Part B. Most recent (preliminary) work on cases with jets

1. Axial Flow Jet

2. Nozzle-Guide-Vane Missile

3. Launch Abort Vehicle with Abort Control Motor Jets

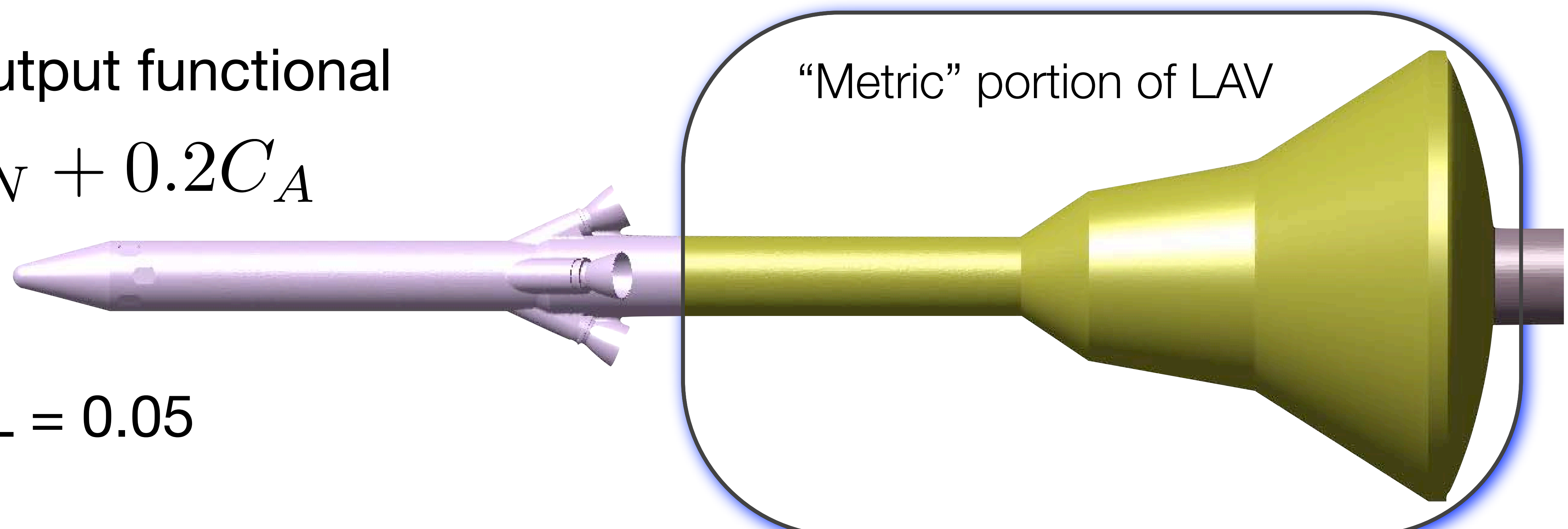
Launch Abort Vehicle (LAV)

- Complex geometry
- Perform aerodynamic analysis for range of operating conditions
  - ➡ M_∞=0.5, 1.1, 1.3
  - ➡ α=-25°, -20°, -12°, -8°, -4°, -2°, 0°, 2°, 4°, 6°
- Goal is to construct an aerodynamic database that satisfies a uniform error tolerance without user supervision

Selected output functional

$$J = C_N + 0.2 C_A$$

TOL = 0.05



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- Complex geometry
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  - ➡ α=-25°, -20°, -12°,

Examine a typical case and discuss local error estimates on uniform meshes

- Goal is to construct an aerodynamic database that satisfies a uniform error tolerance without user supervision

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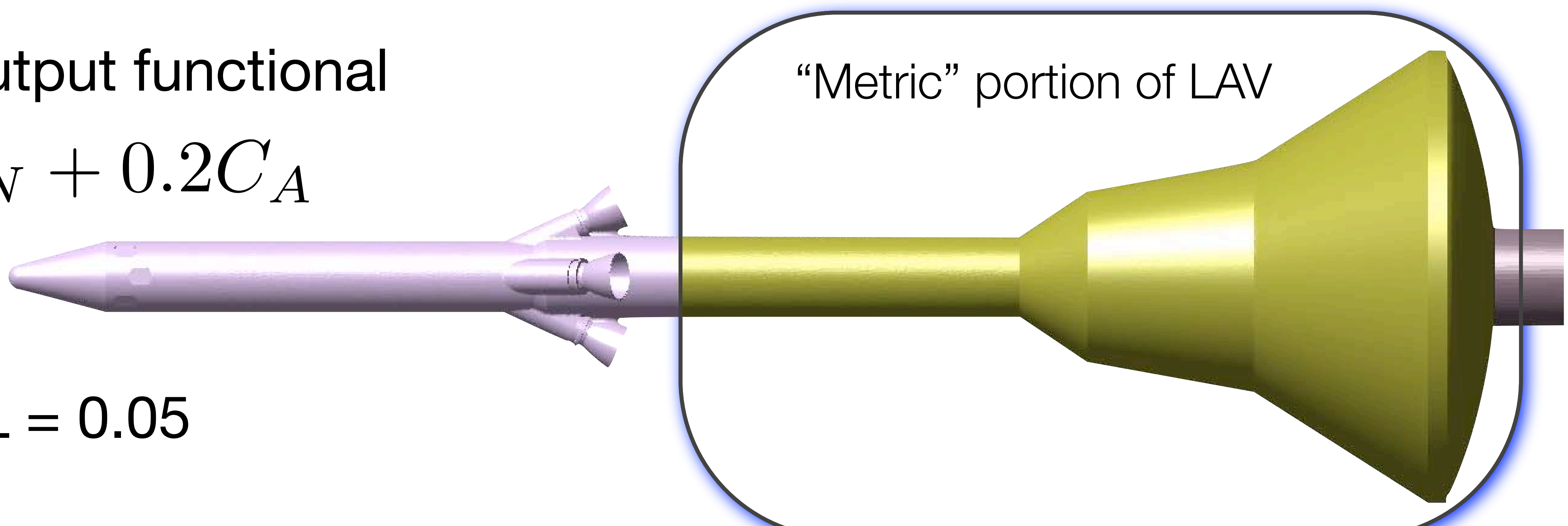
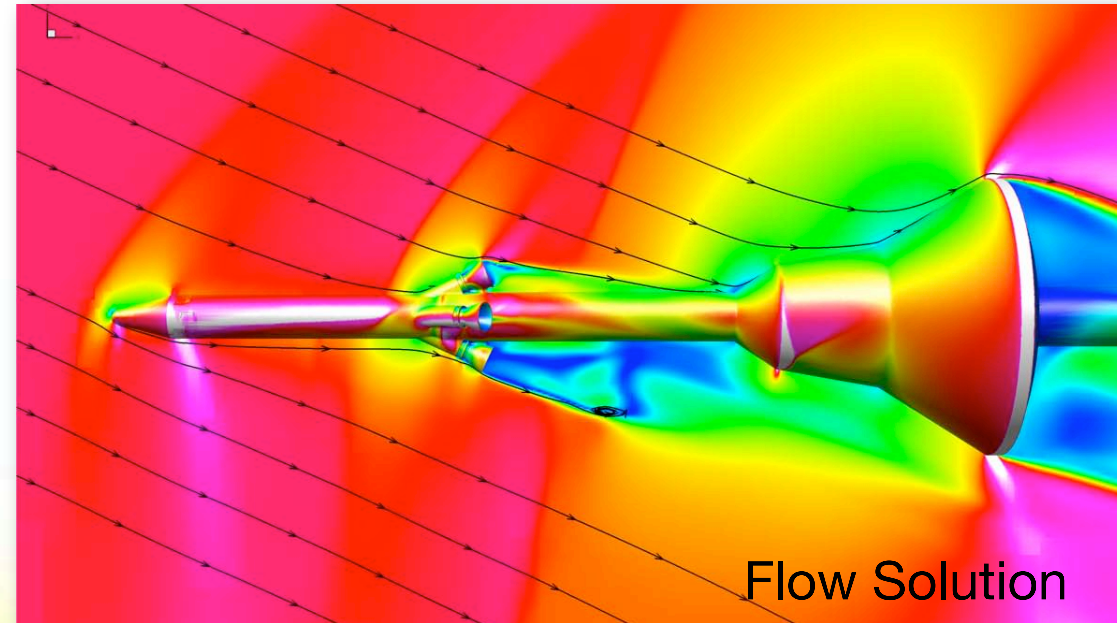
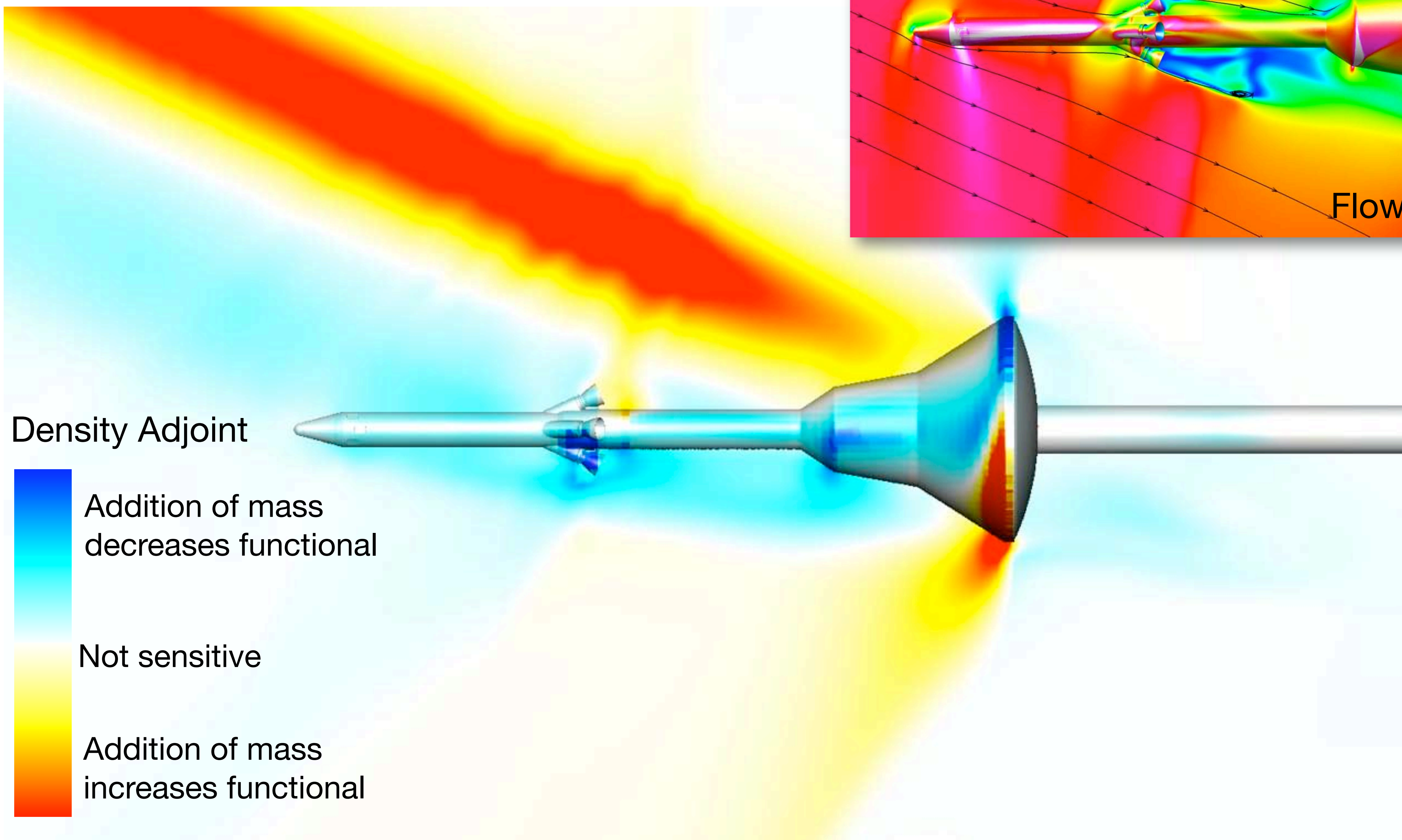


Adjoint Solution

M_∞=1.1, α=-25°

$$J = C_N + 0.2C_A$$

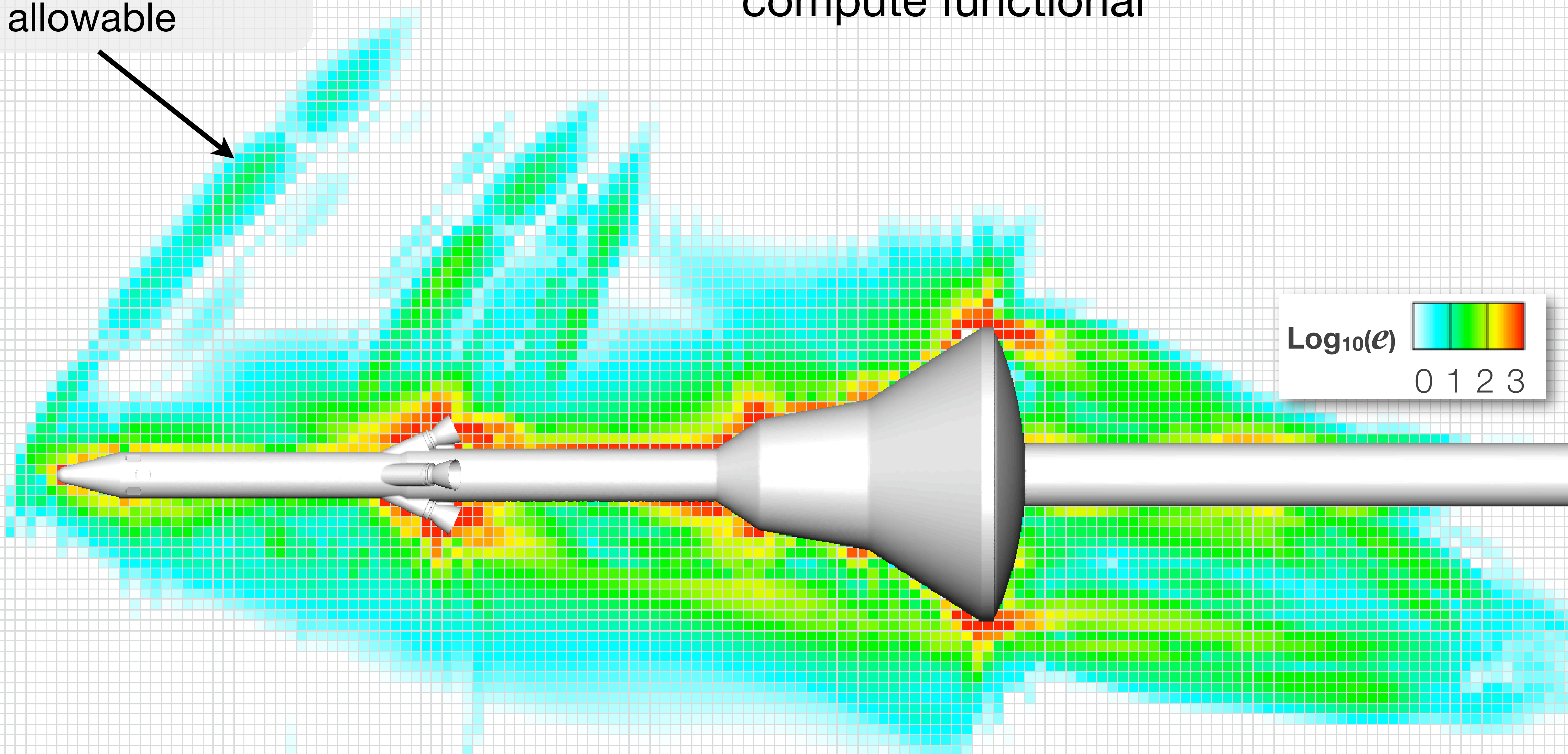




Local Error Estimates

Adapt any cell that contributes more error to the functional than is allowable

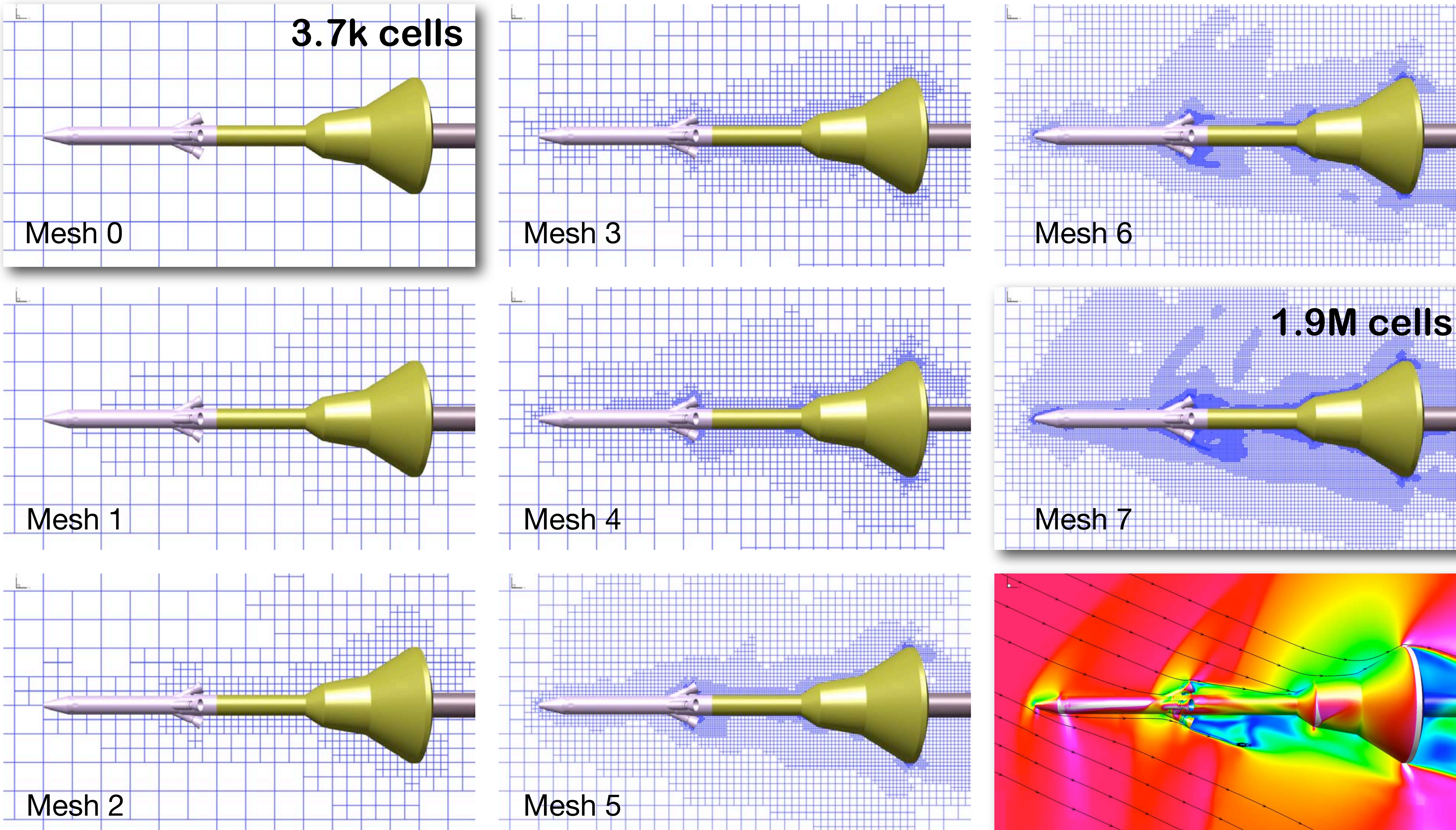
- Error map shows relative importance of flow features
- Determines cell-size to accurately compute functional



M∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05

Example Mesh Evolution

$M_\infty = 1.1$, $\alpha = -25°$



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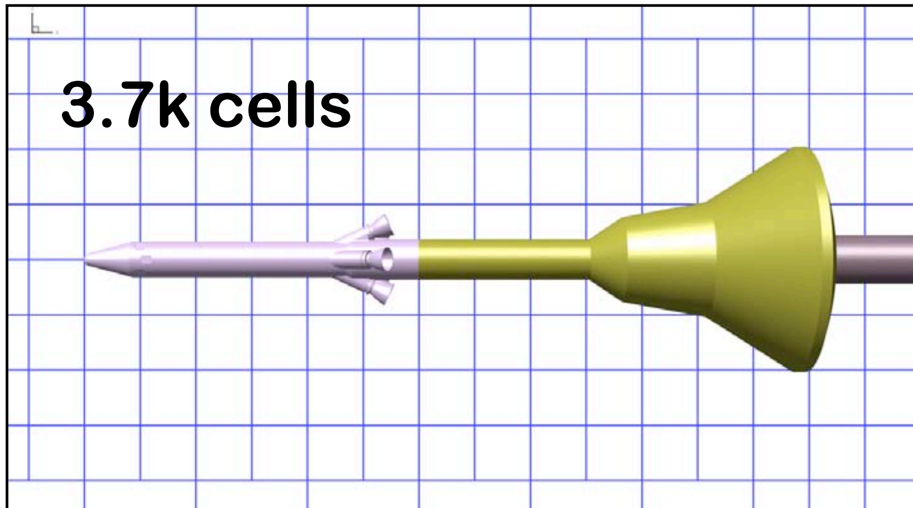
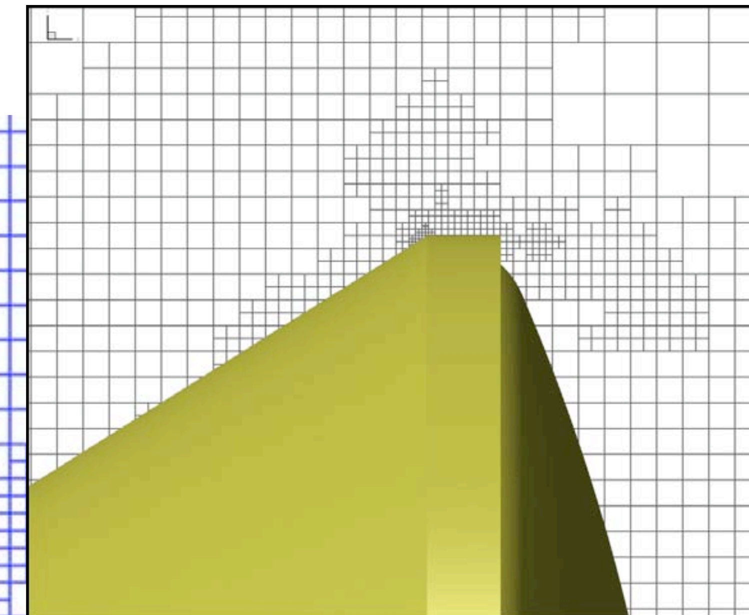
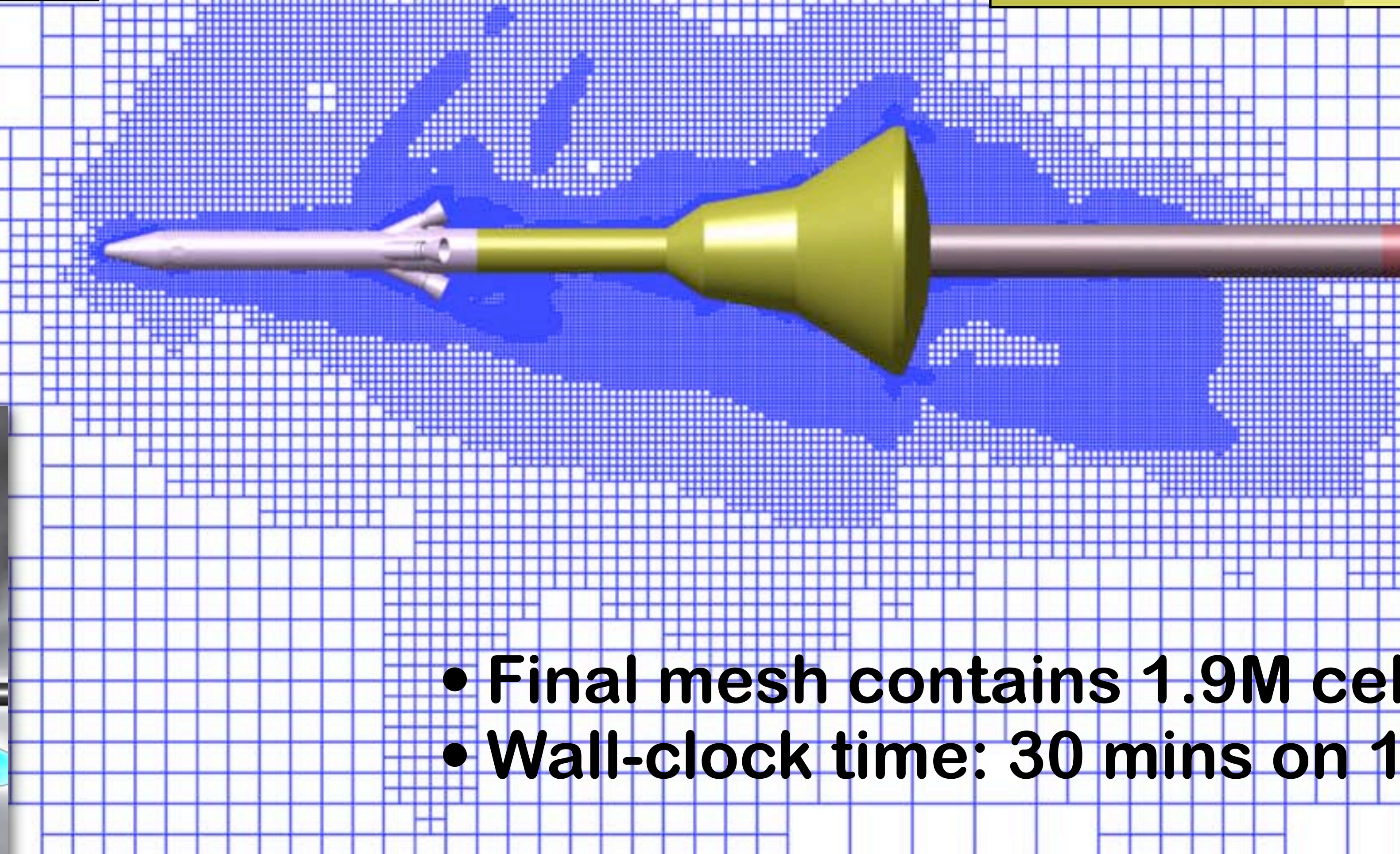


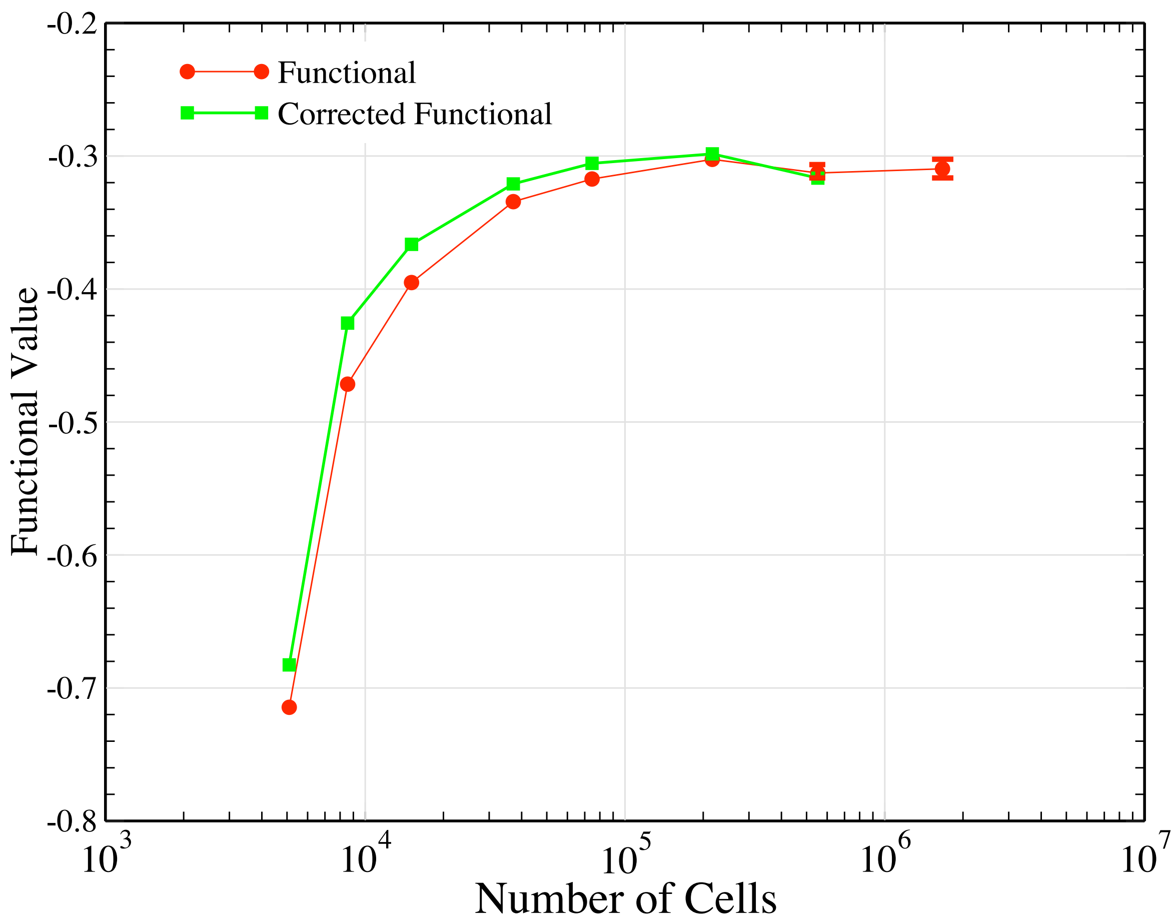


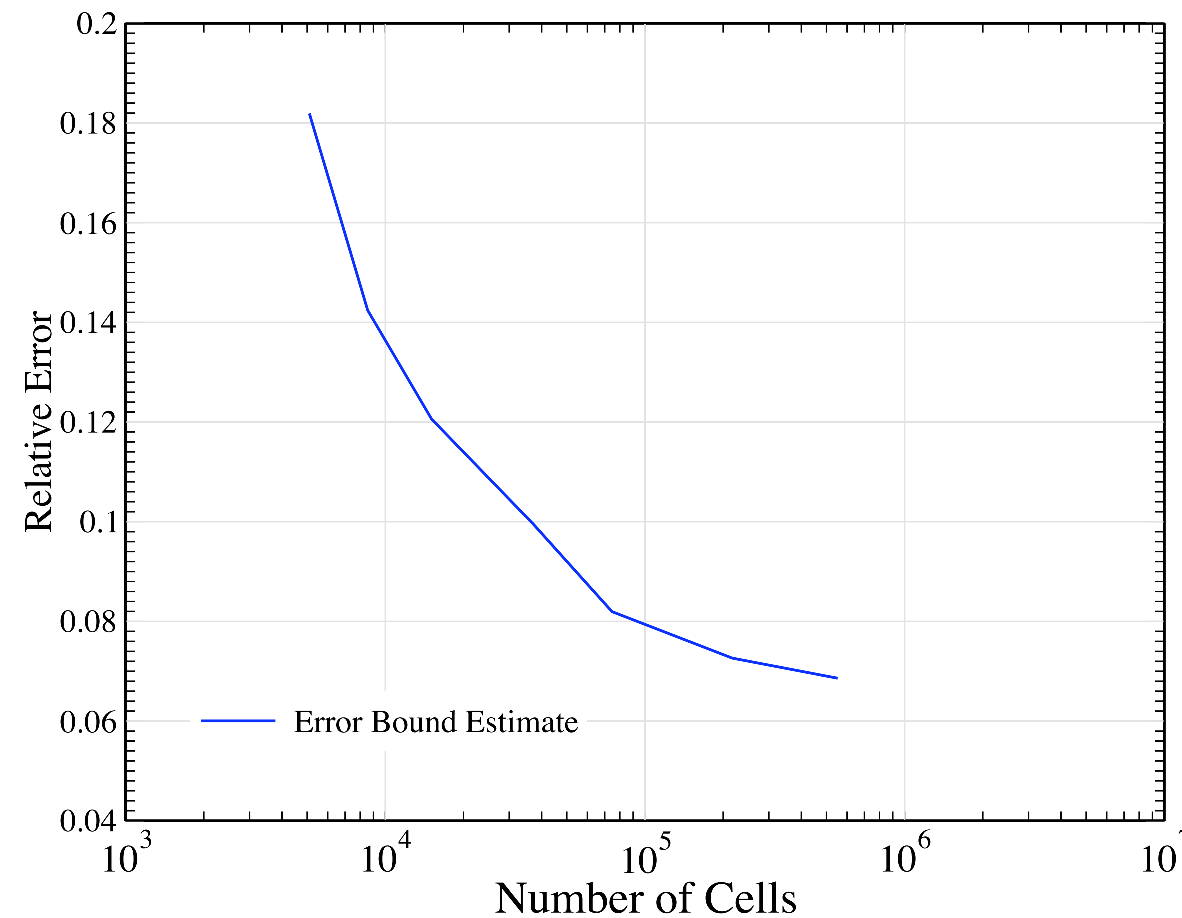


- **Final mesh contains 1.9M cells**
- **Wall-clock time: 30 mins on 16 CPUs**

Functional Convergence

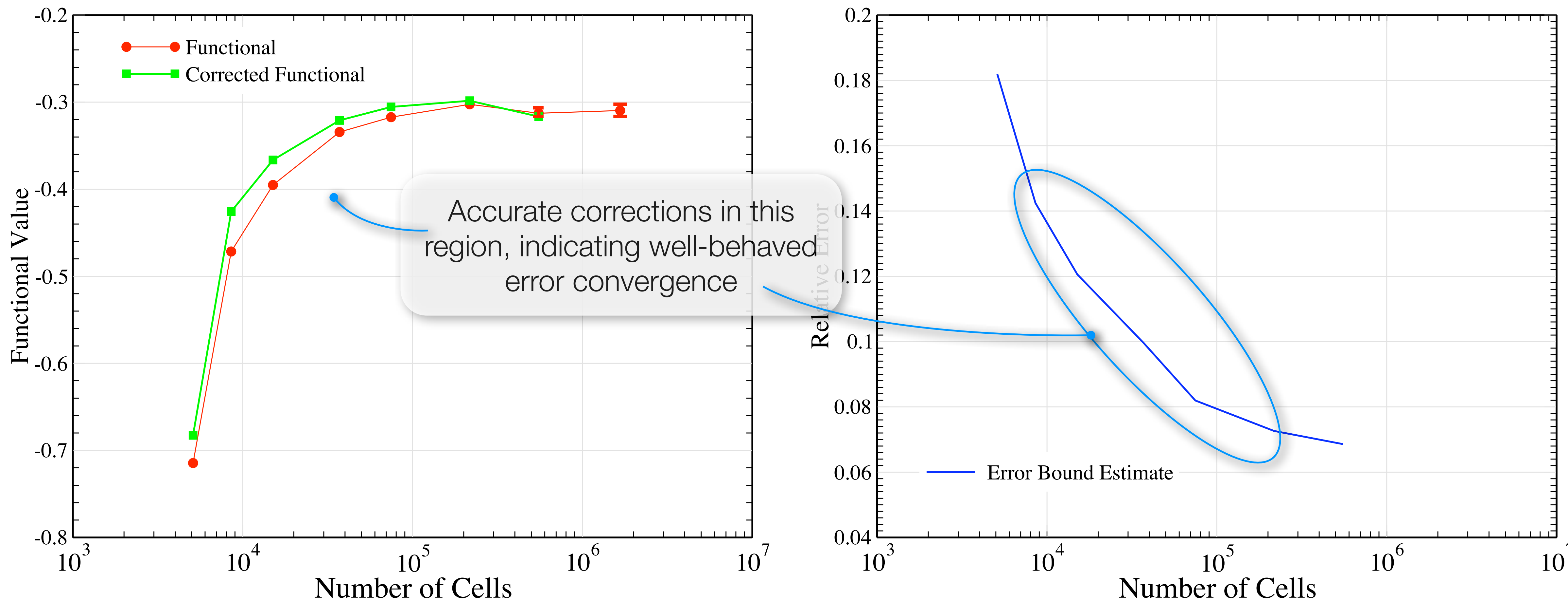
M_∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05





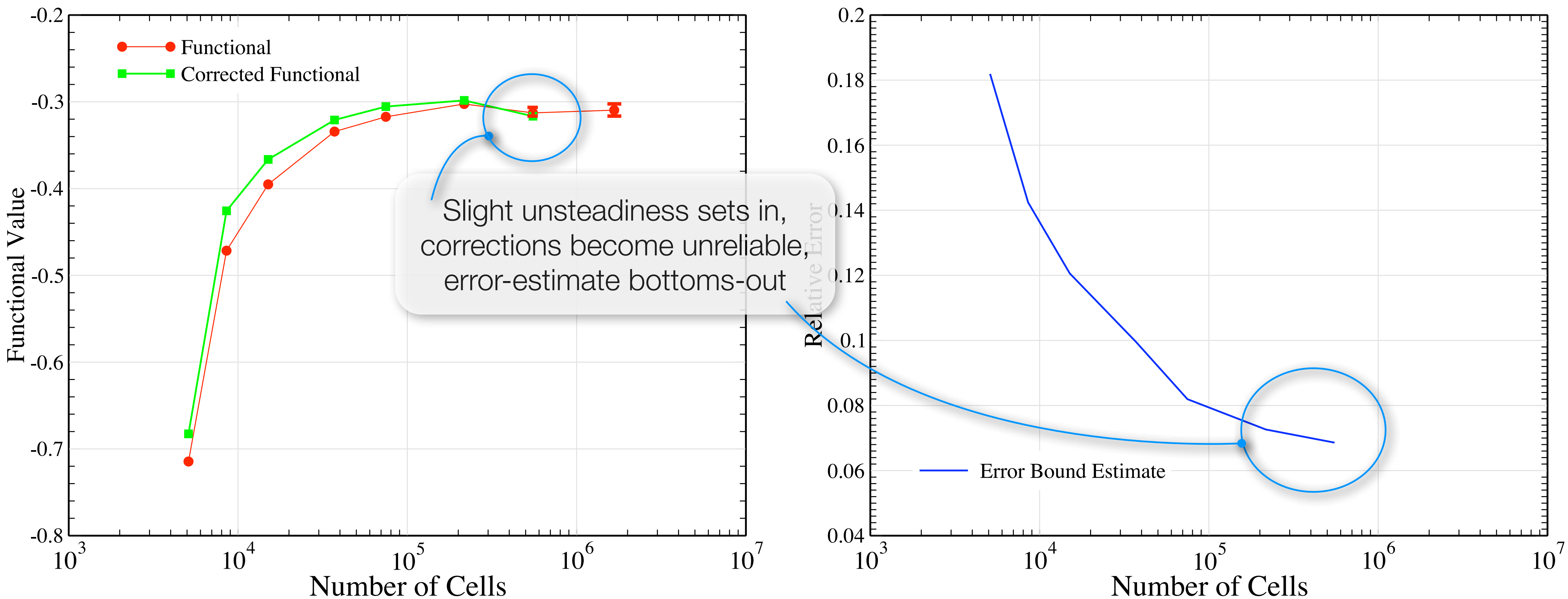
Functional Convergence

M∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05



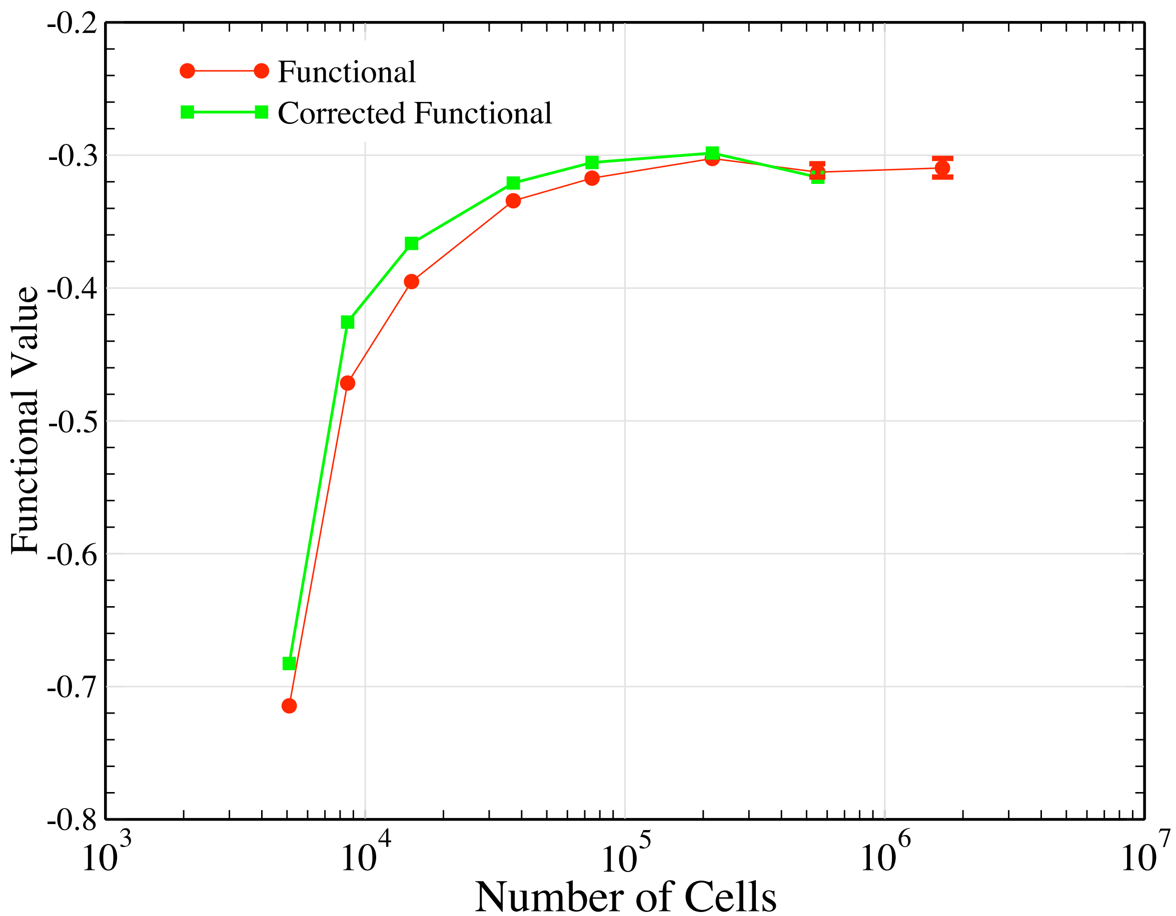
Functional Convergence

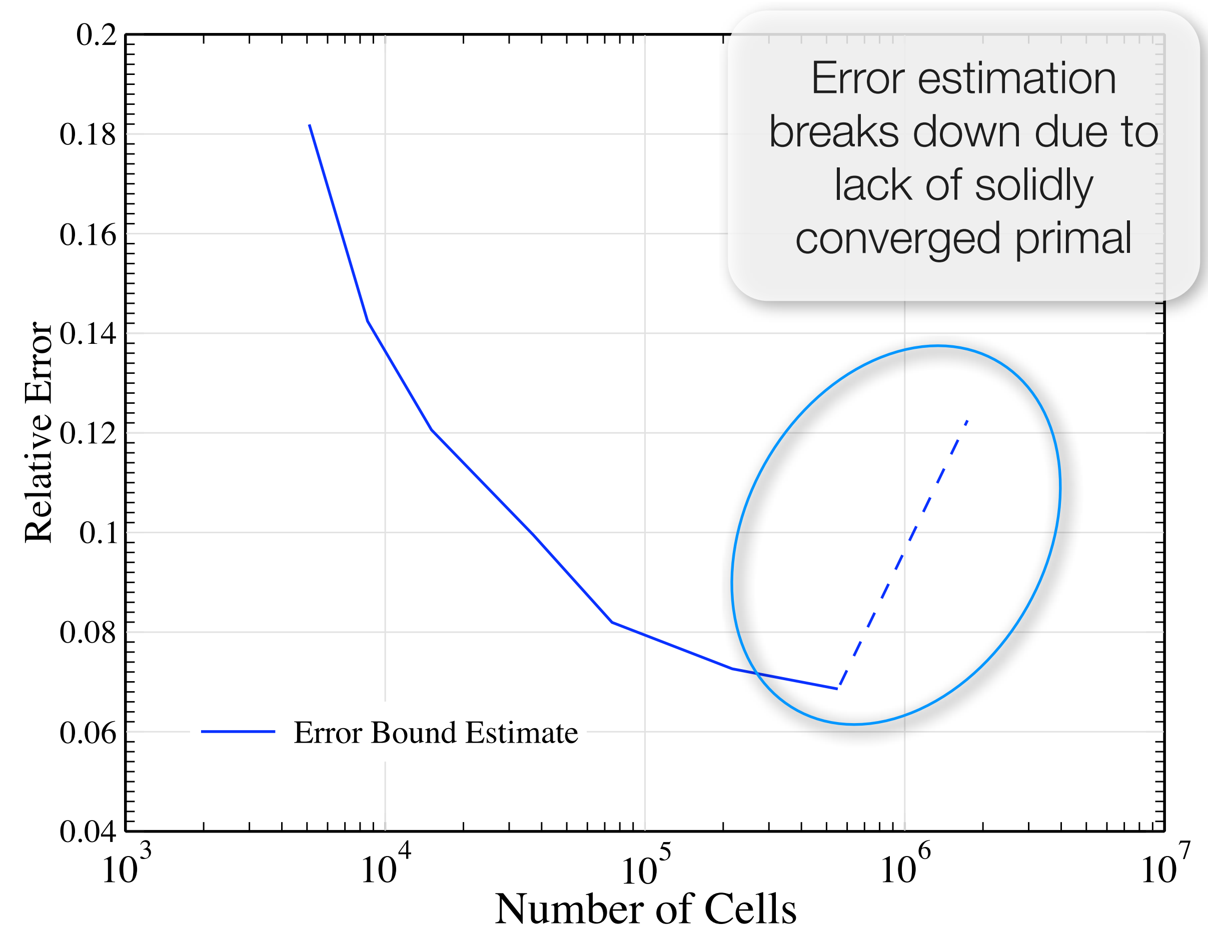
M∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05



Functional Convergence

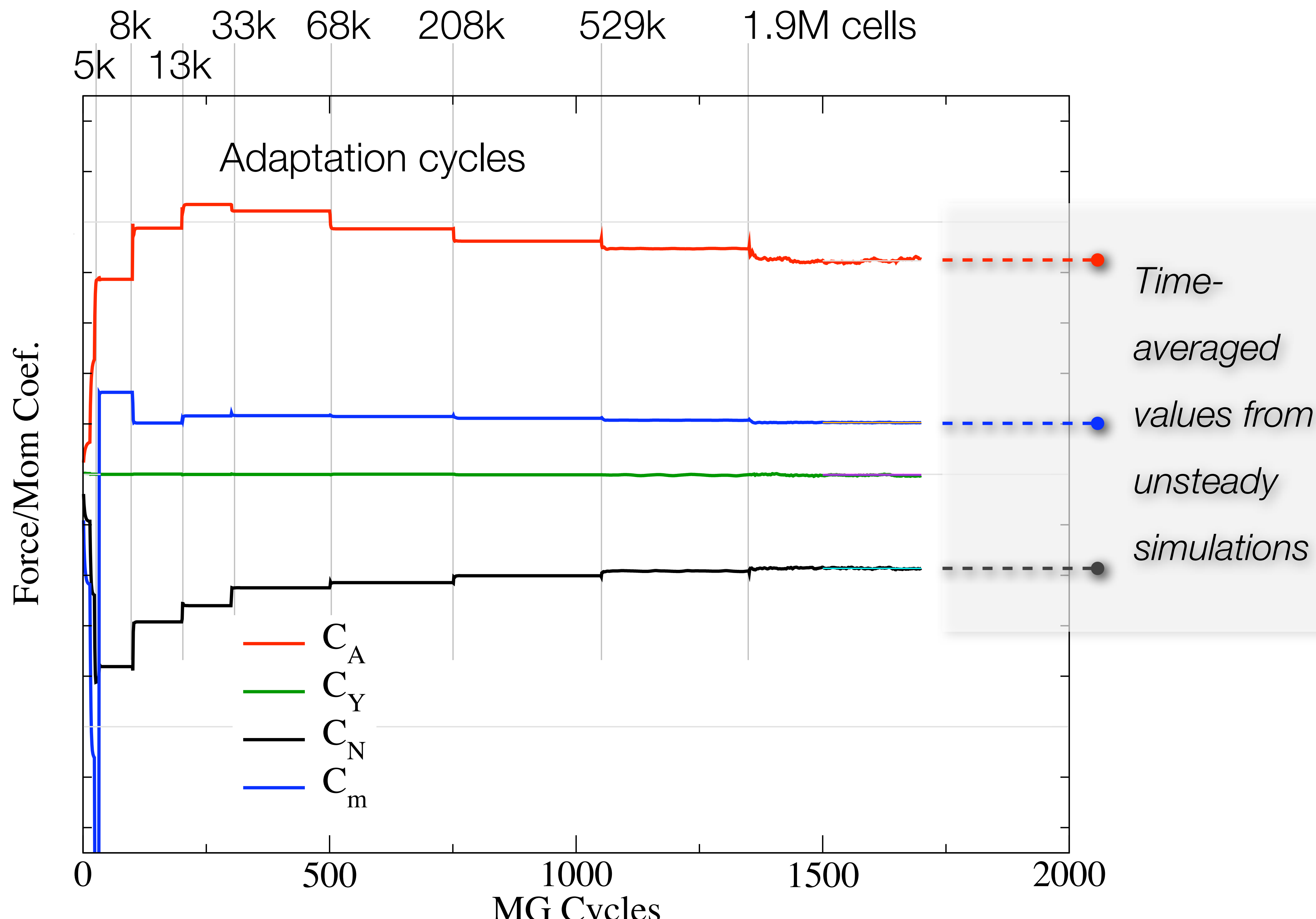
M_∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05



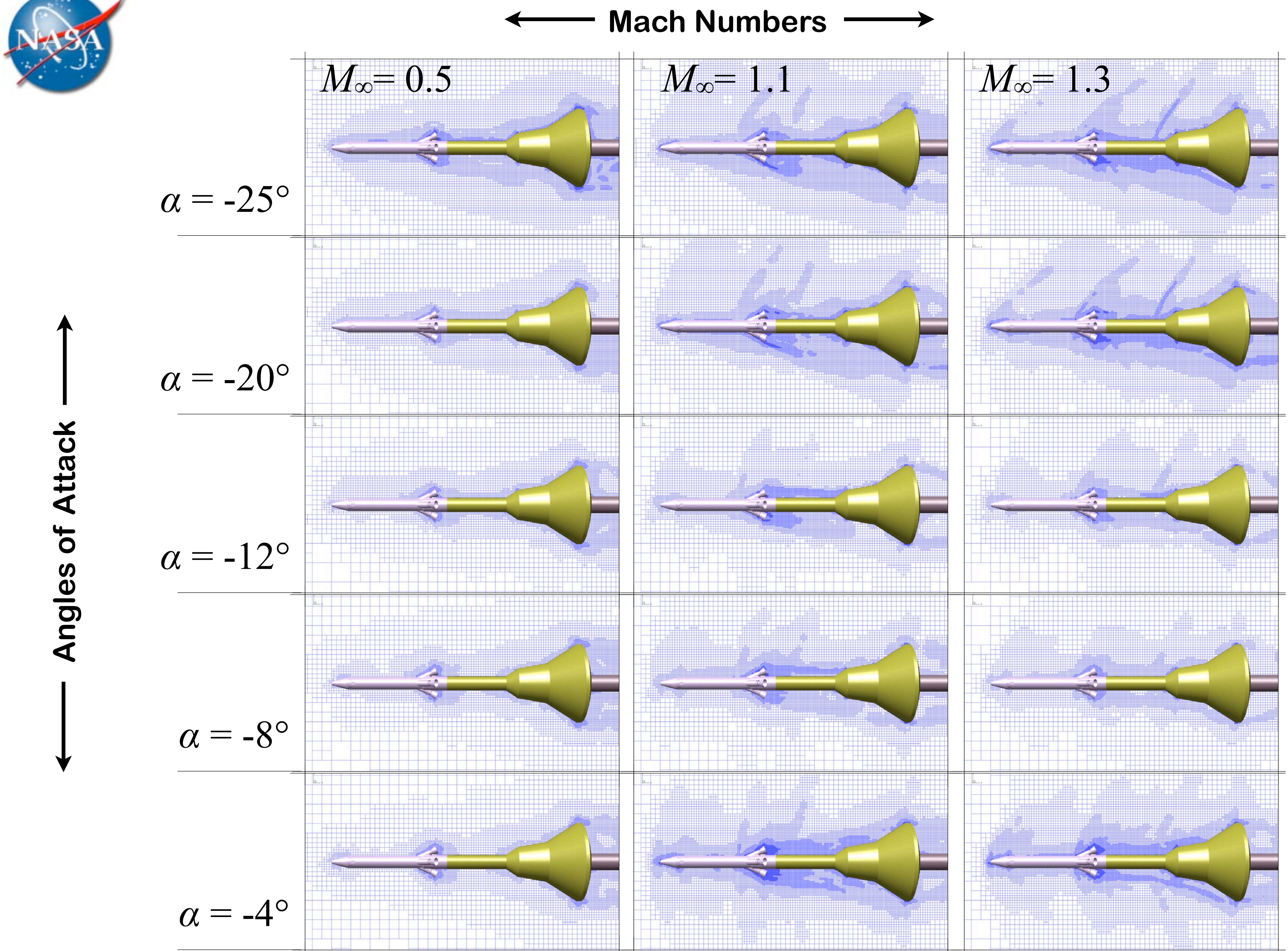


- Convergence of functional, correction and error-bound estimate provides insight into onset of “unsteadiness” (noise due to incomplete convergence)

Functional Convergence

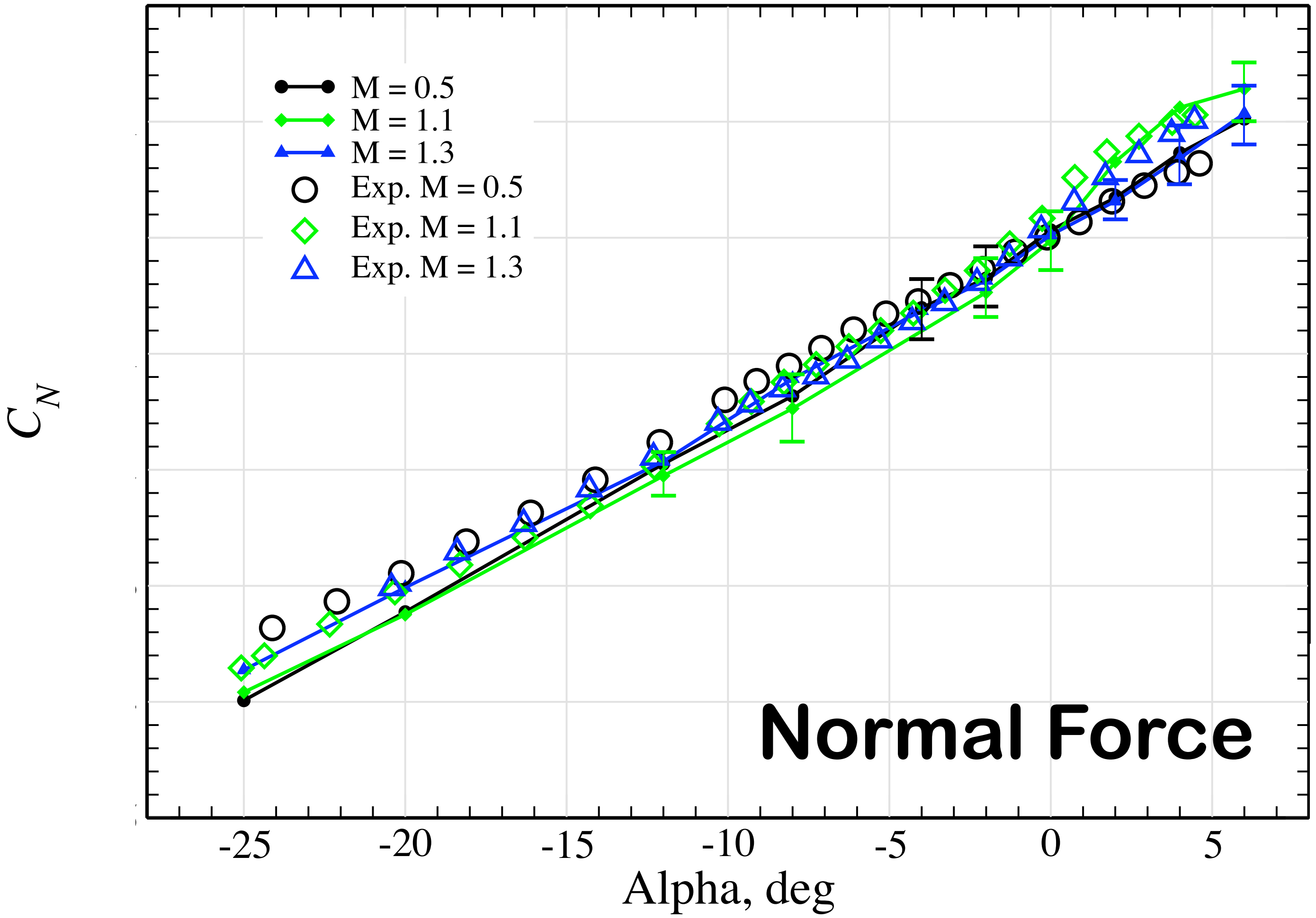


M∞=1.1, α=-25°, $J = C_N + 0.2C_A$, TOL = 0.05



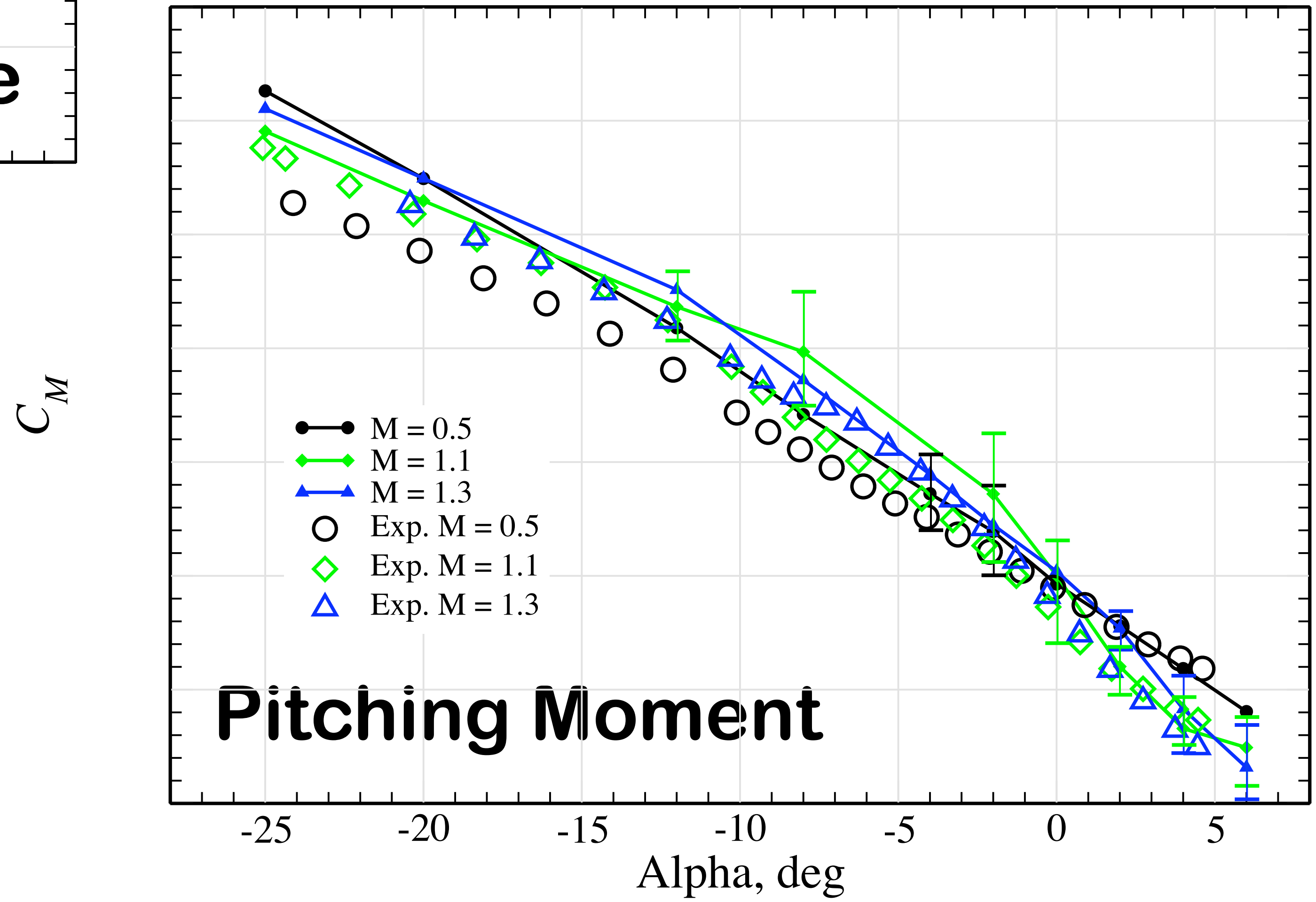
Mach Numbers
$M_\infty = 0.5$
$M_\infty = 1.1$
$M_\infty = 1.3$
Angles of Attack
$\alpha = -25°$
$\alpha = -20°$
$\alpha = -12°$
$\alpha = -8°$
$\alpha = -4°$

Comparison with Experiment



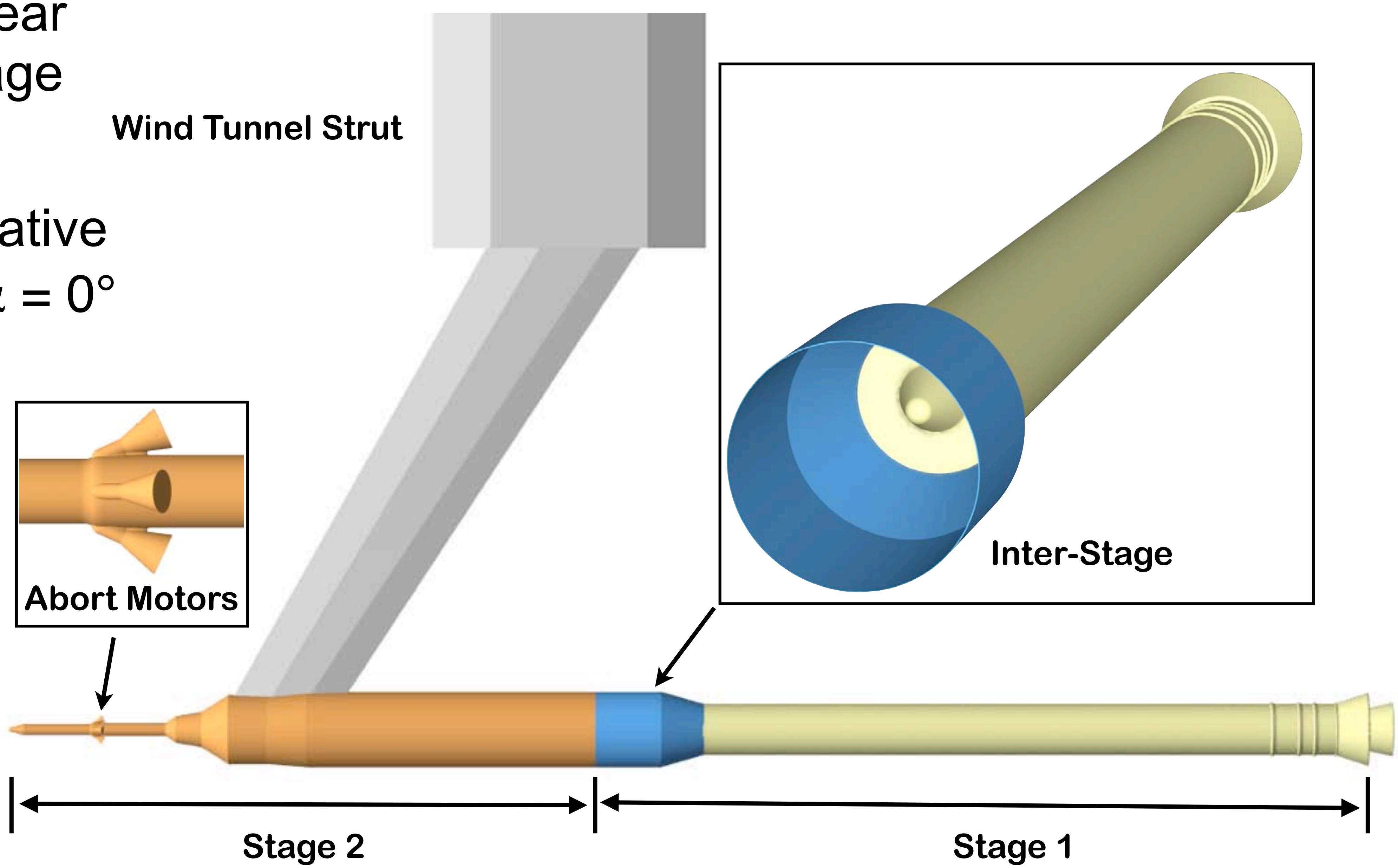
- Run matrix involved 30 cases
- General agreement with experimental values for normal and axial forces, and pitching moment

- Error bars indicate level of noise in functional due to incomplete flow convergence
- Indication of where unsteady analysis is required

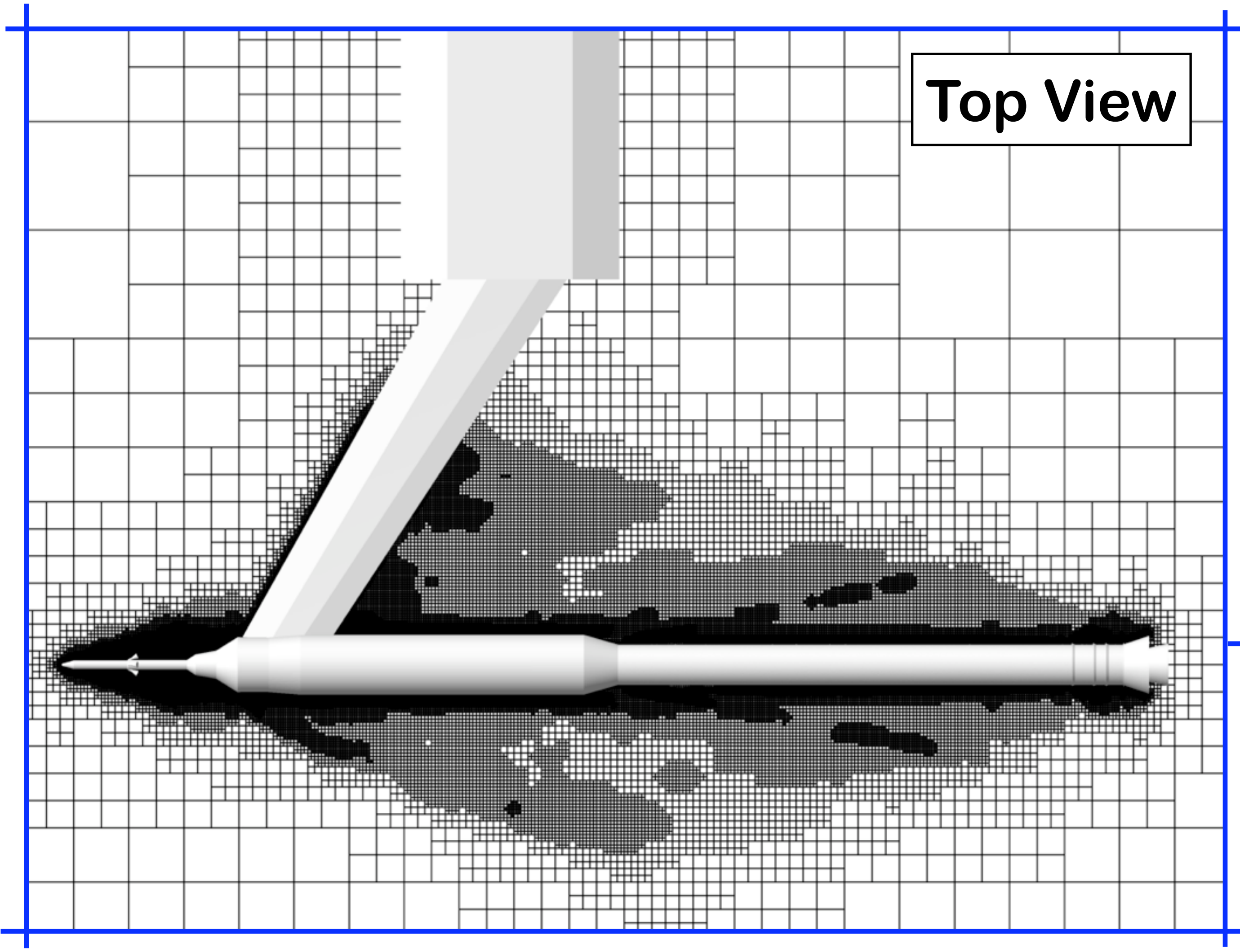


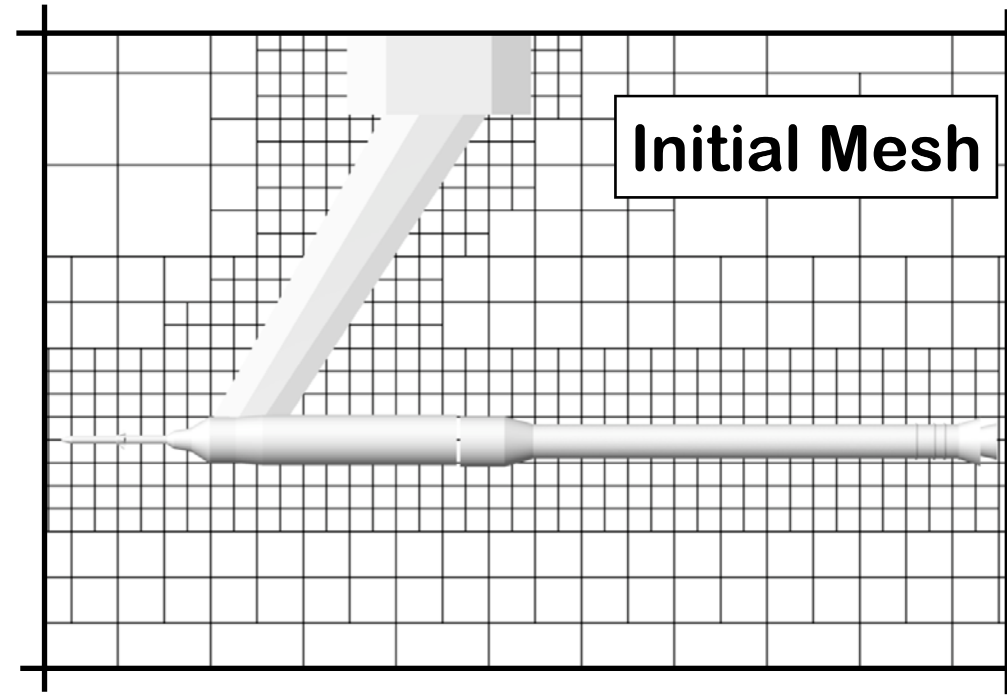
Launch-Vehicle Stage Separation

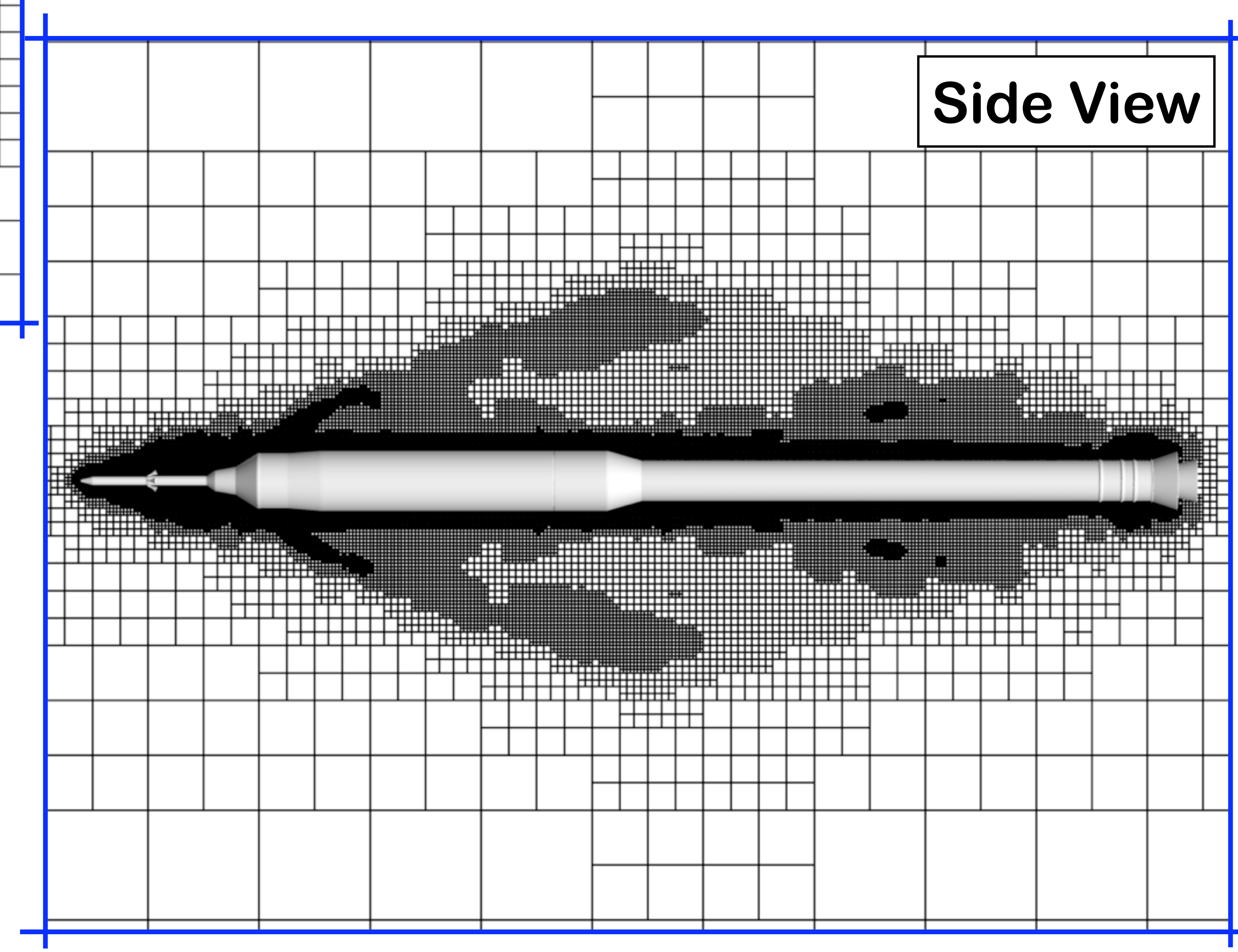
- Parametric study of aerodynamic performance for various separation distances and strut interference effects
- Roughly 25 configurations at two angles of attack with fixed Mach number
- Functional is a linear combination of Stage 1 and 2 drag
- Present representative case at $M_\infty = 4.5$, $\alpha = 0°$
- Decreasing Threshold: $\lambda = 16, \ldots, 1$
- Uniform error tolerance of 0.1



Near-Body Mesh Views

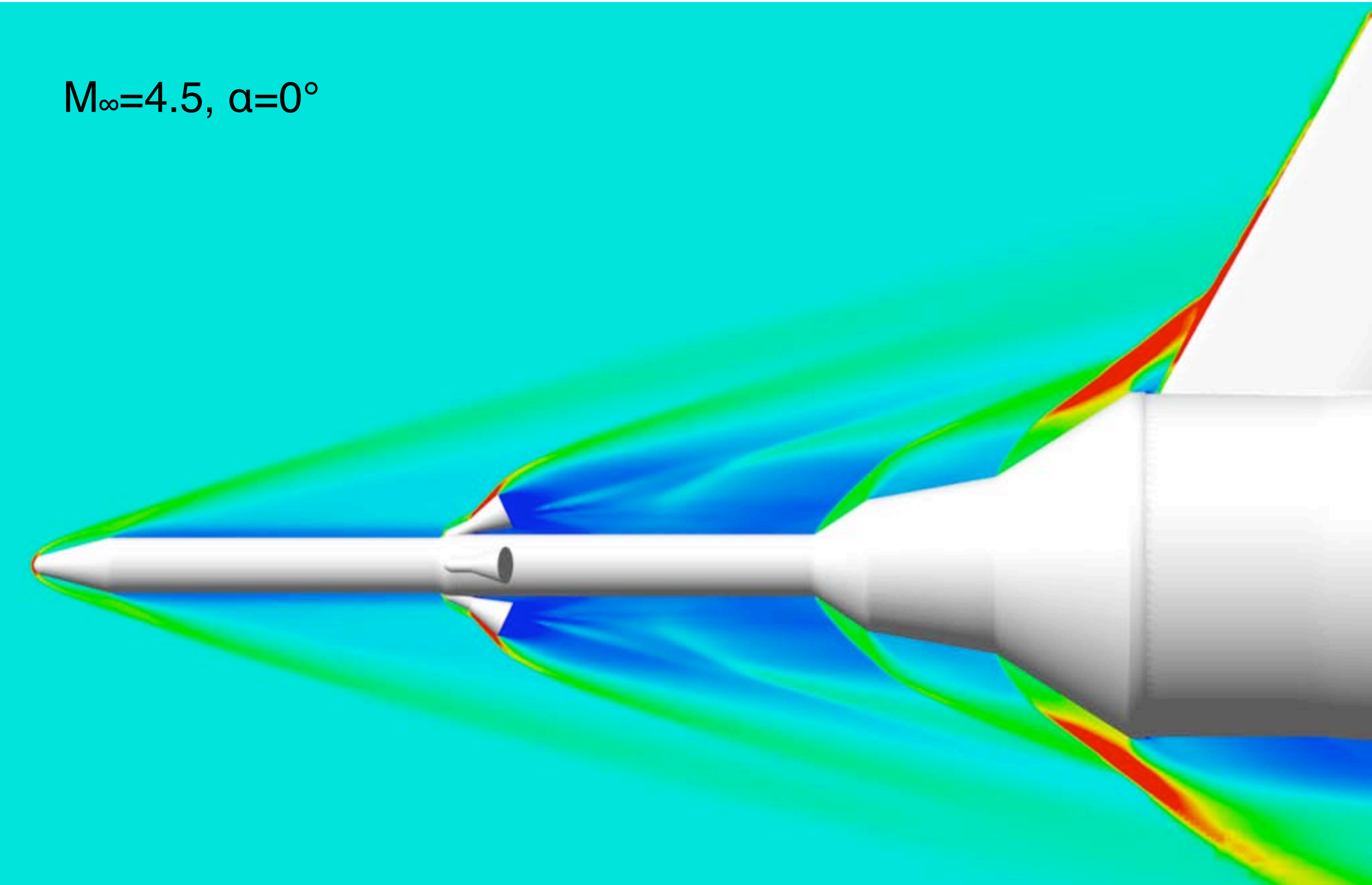




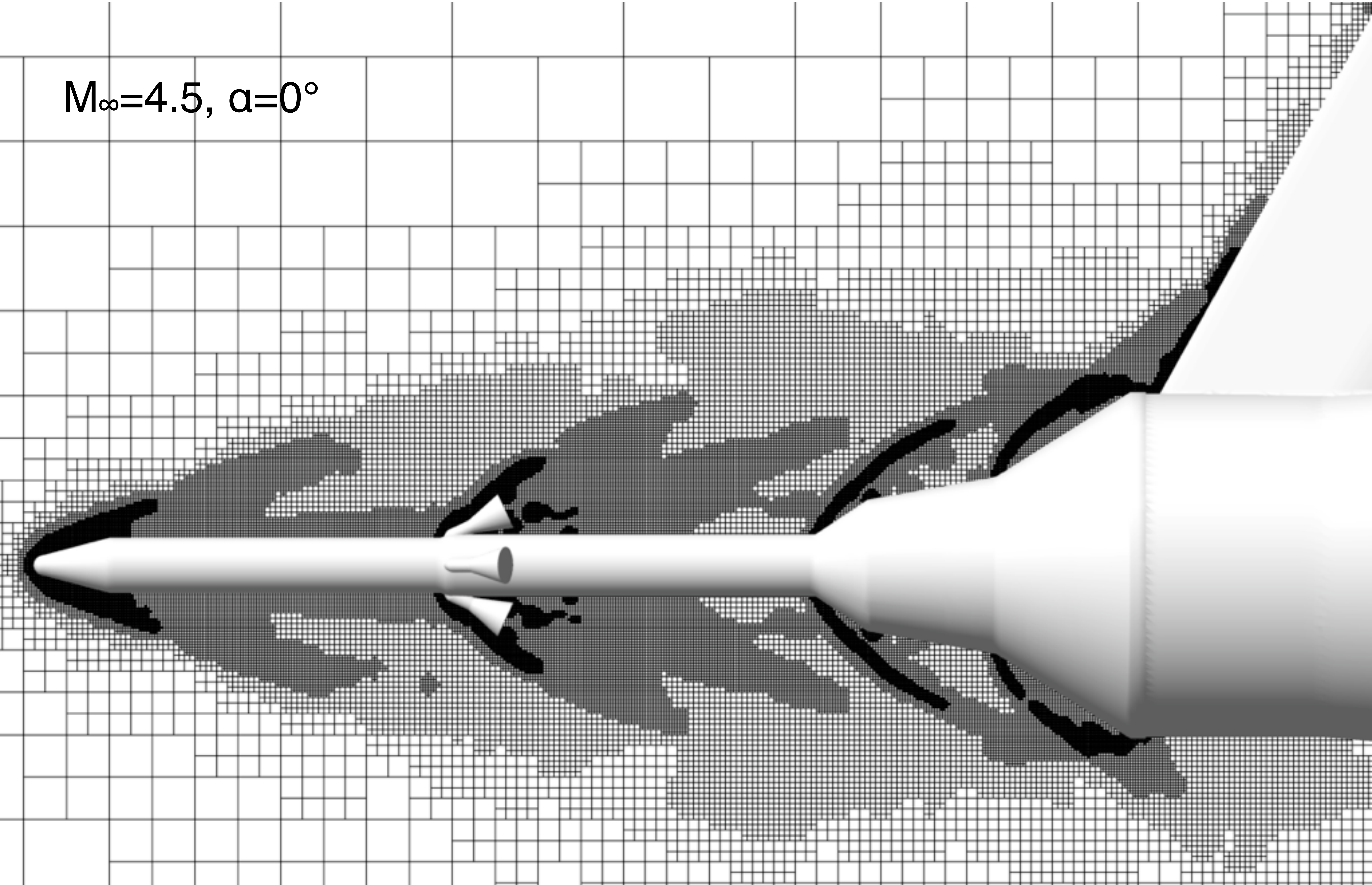


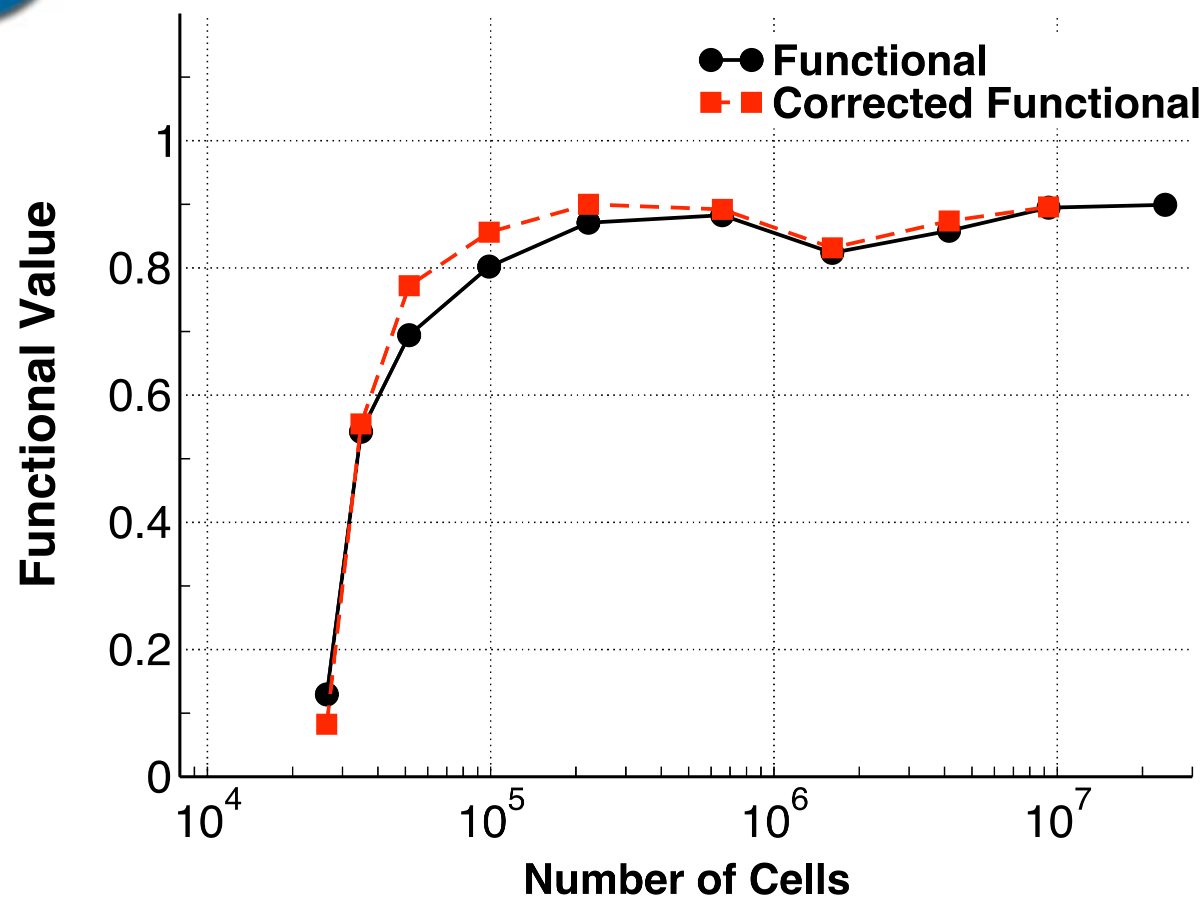
- Initial mesh contains only 13k cells
- Final meshes contain between 8M to 20M cells

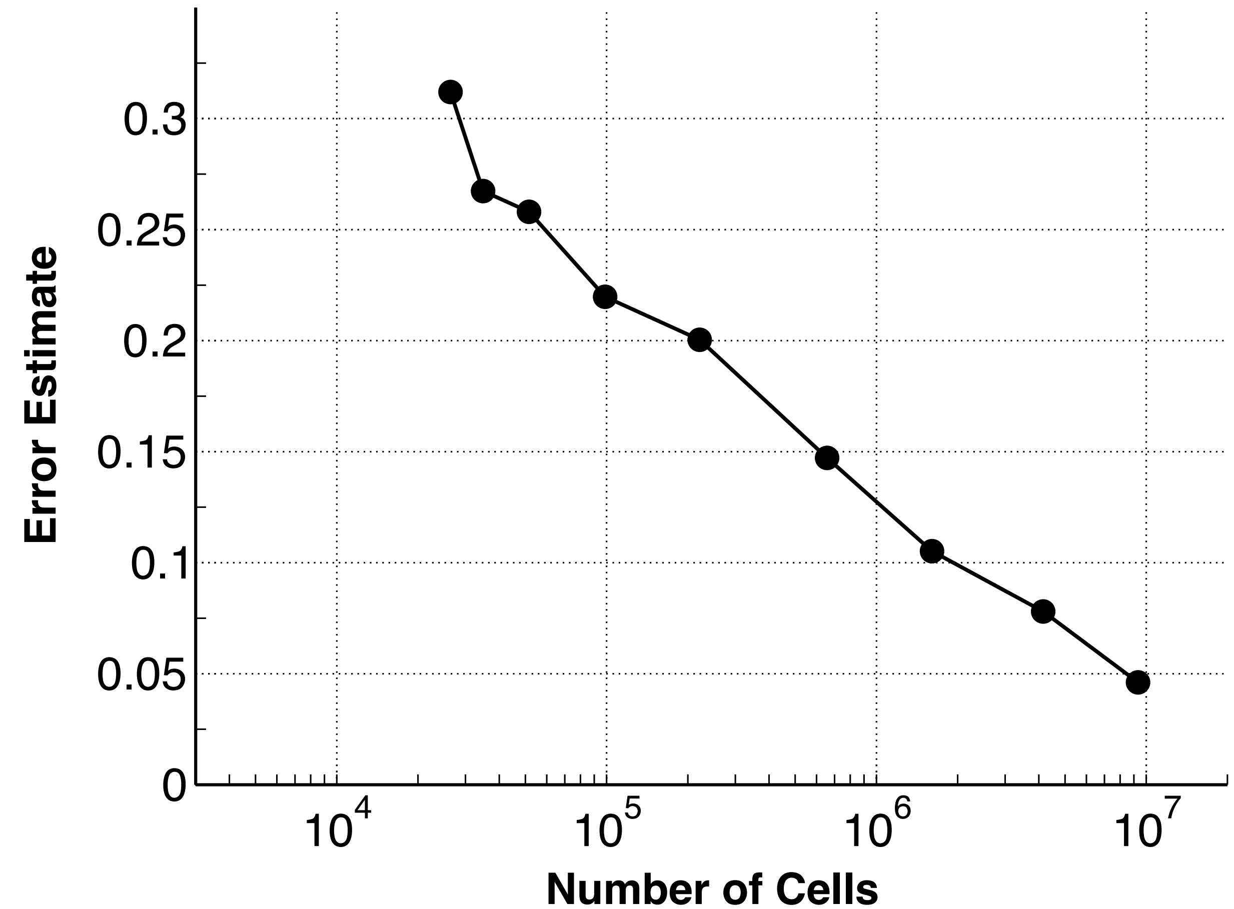
Pressure Contours



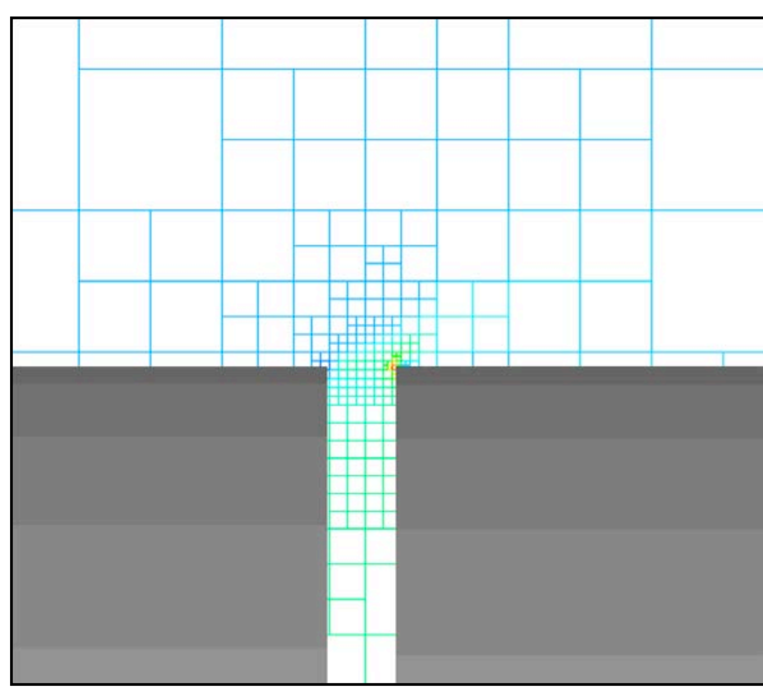
Pressure Contours



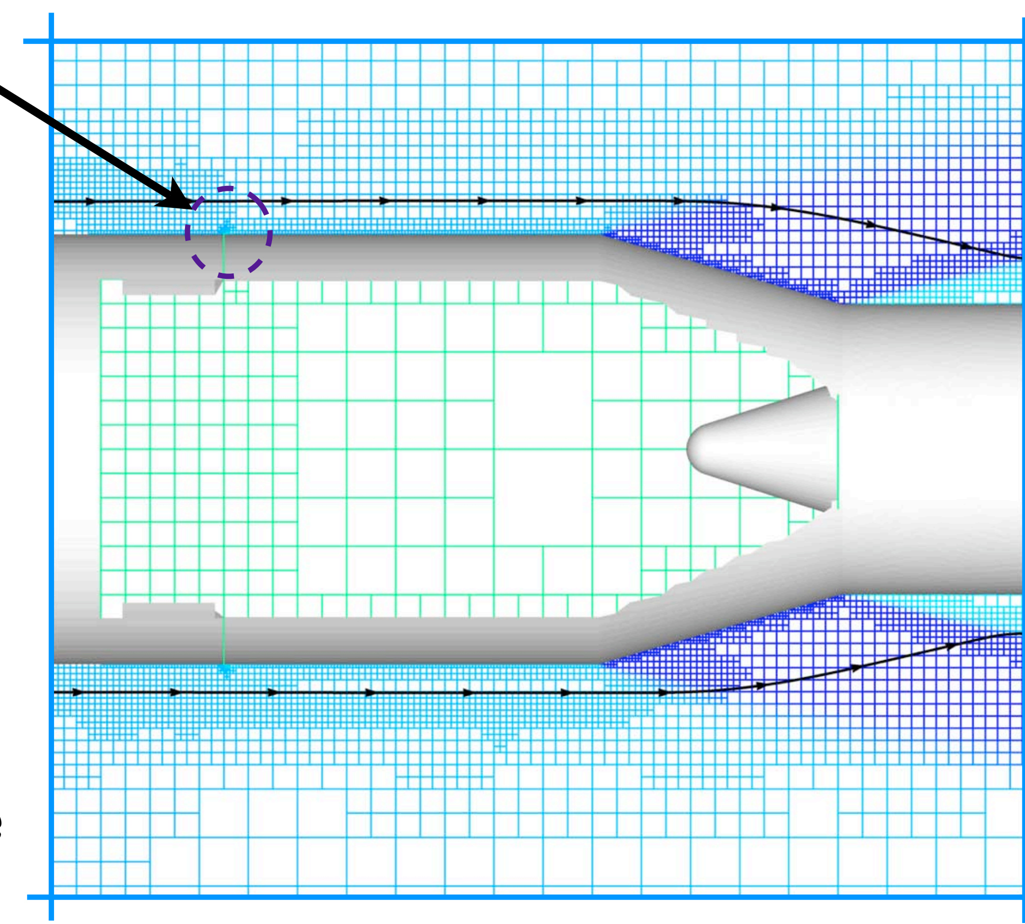




- Minimal refinement of inter-stage region
- Gap is highly refined
- Overall, excellent convergence of functional and error estimate

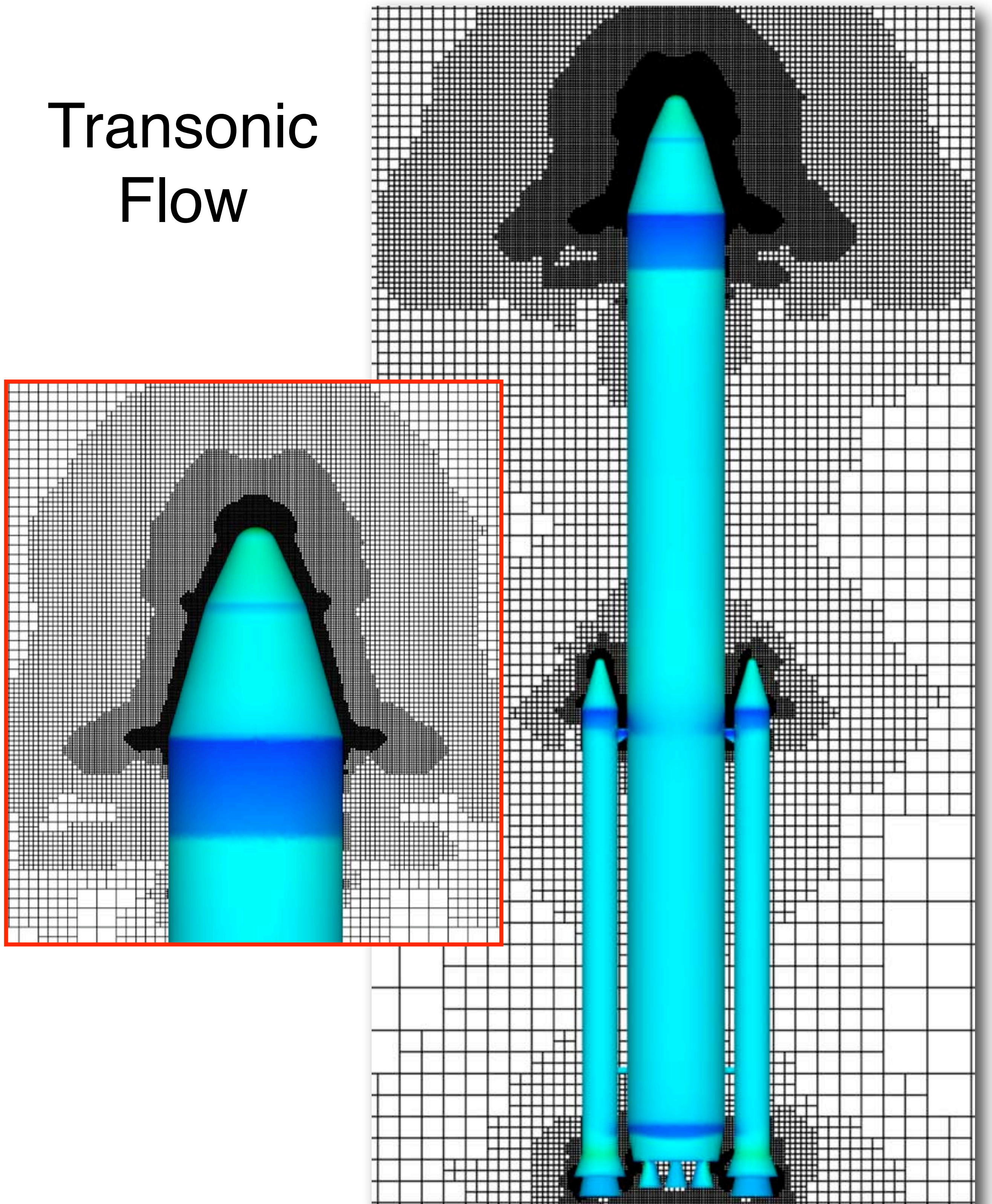


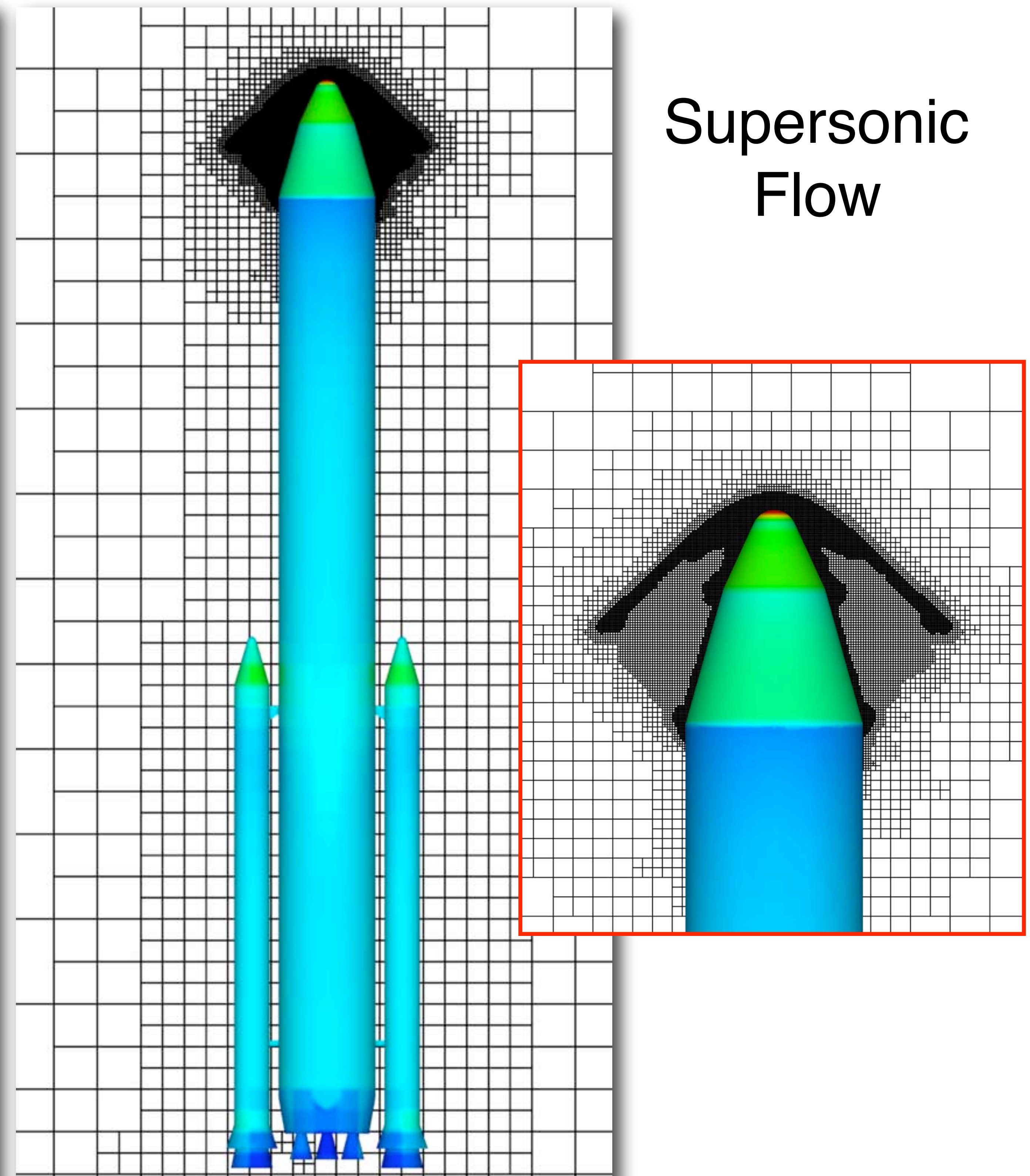
Cutaway view of inter-stage



Launch Vehicle
Functional: Axial force on nose fairing

Transonic Flow

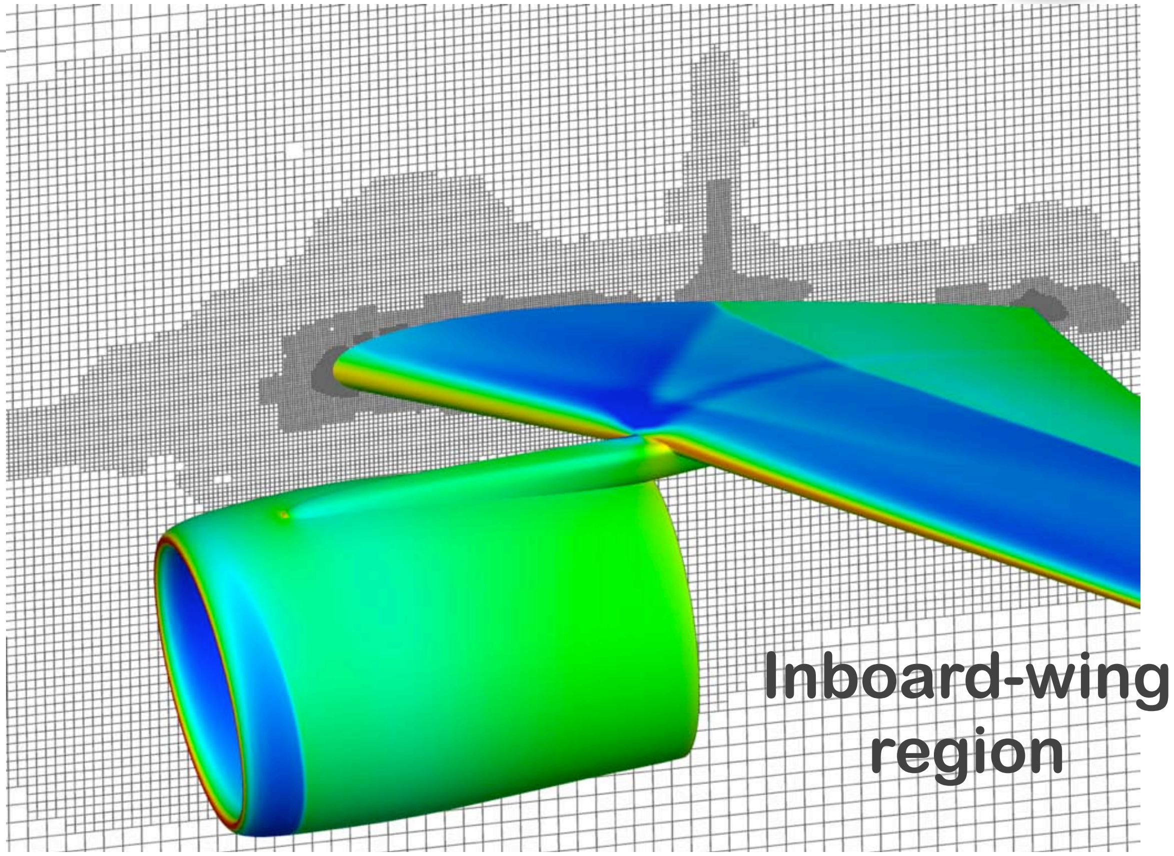


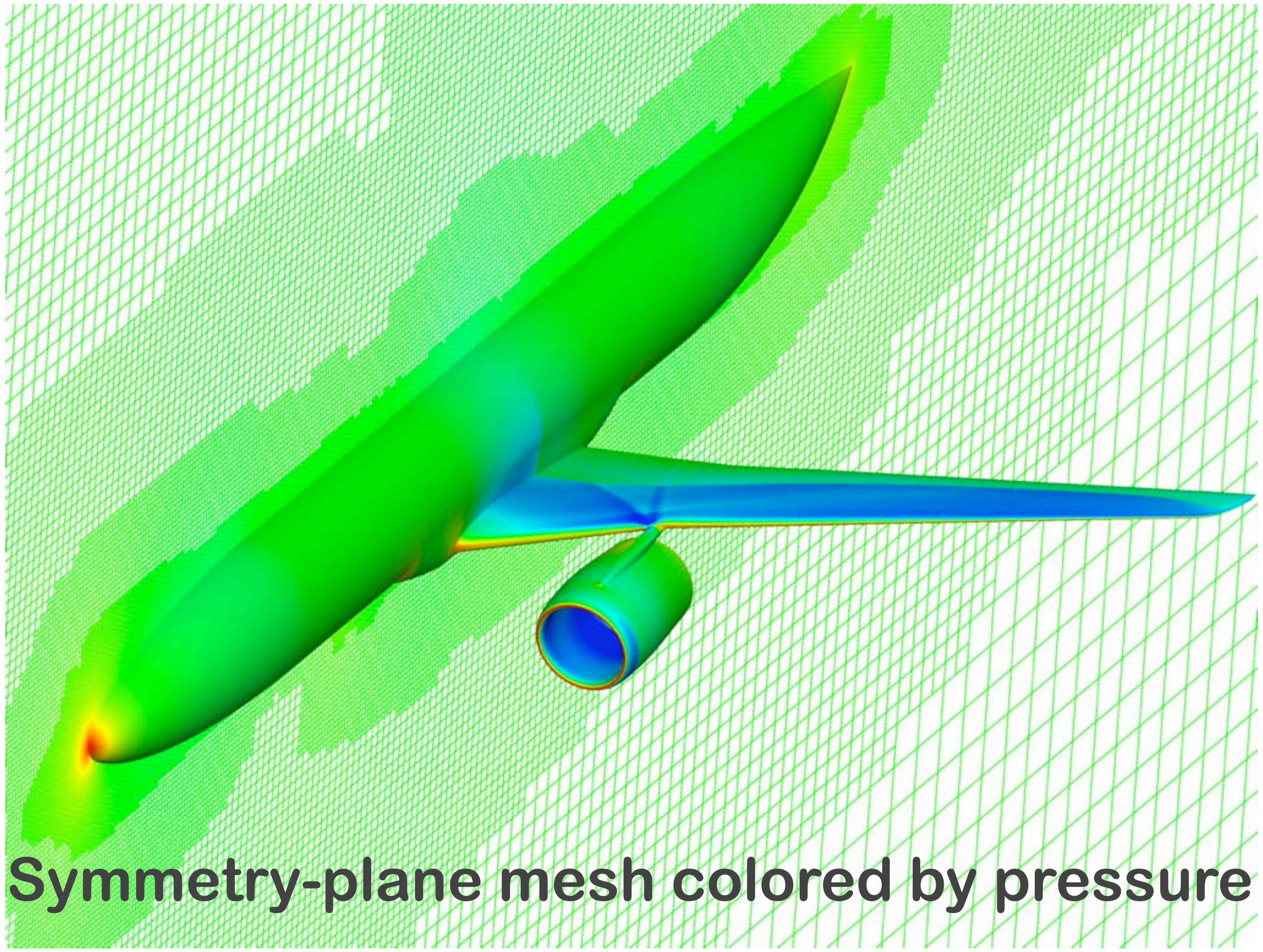


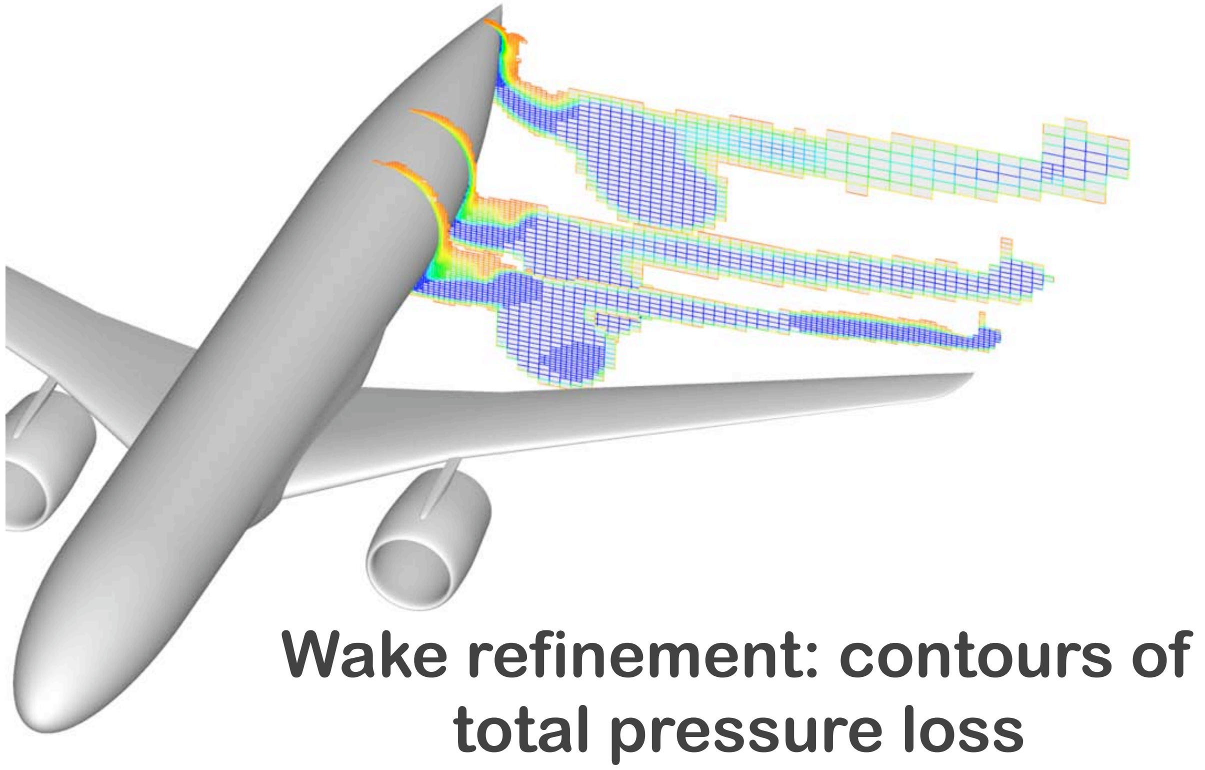
Supersonic Flow

Transport Aircraft

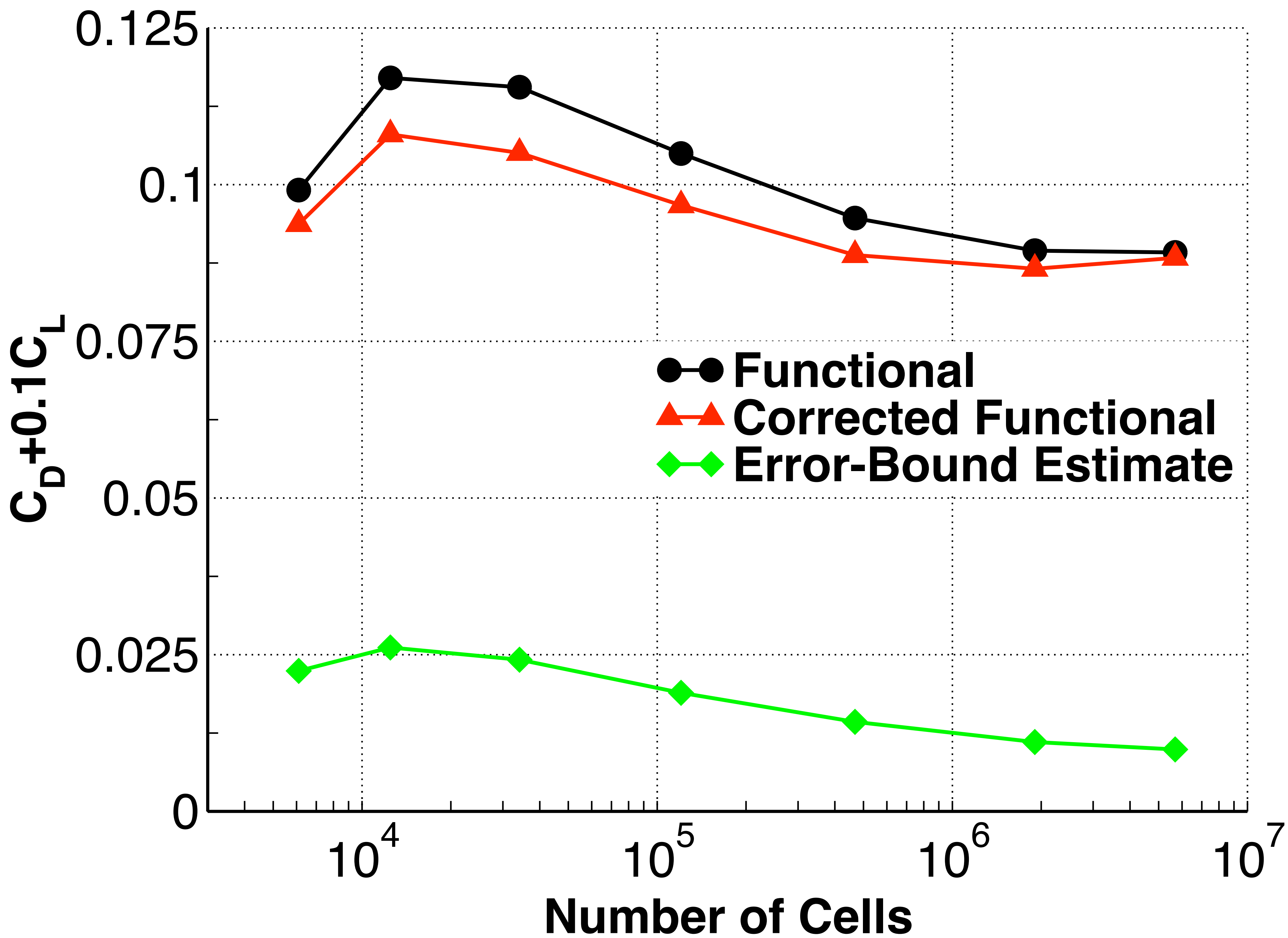
- Transonic flow: $M_\infty = 0.8$, $\alpha = 2°$
- Functional: $C_D + 0.1C_L$
- Initial mesh ~6k cells
- Final mesh ~5.7M cells (6 adaptations)







Transport Aircraft
Functional Convergence



Quiet Supersonic Cruise

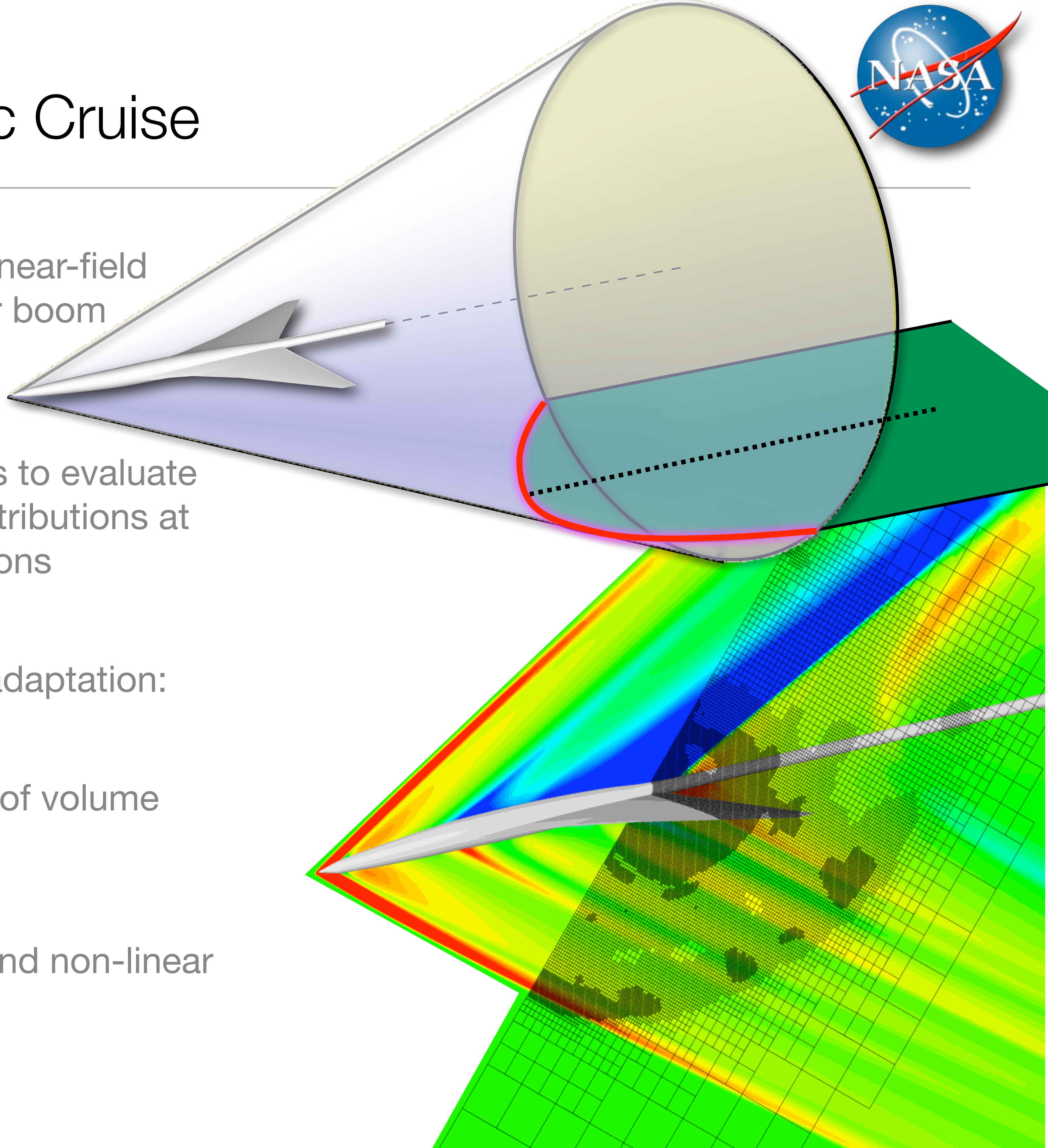
- Goal is determination of near-field pressure distributions for boom prediction ("N"-waves)

- Use adjoint error analysis to evaluate accuracy of pressure distributions at specified "sensor" locations

- Ideal problem for mesh adaptation:

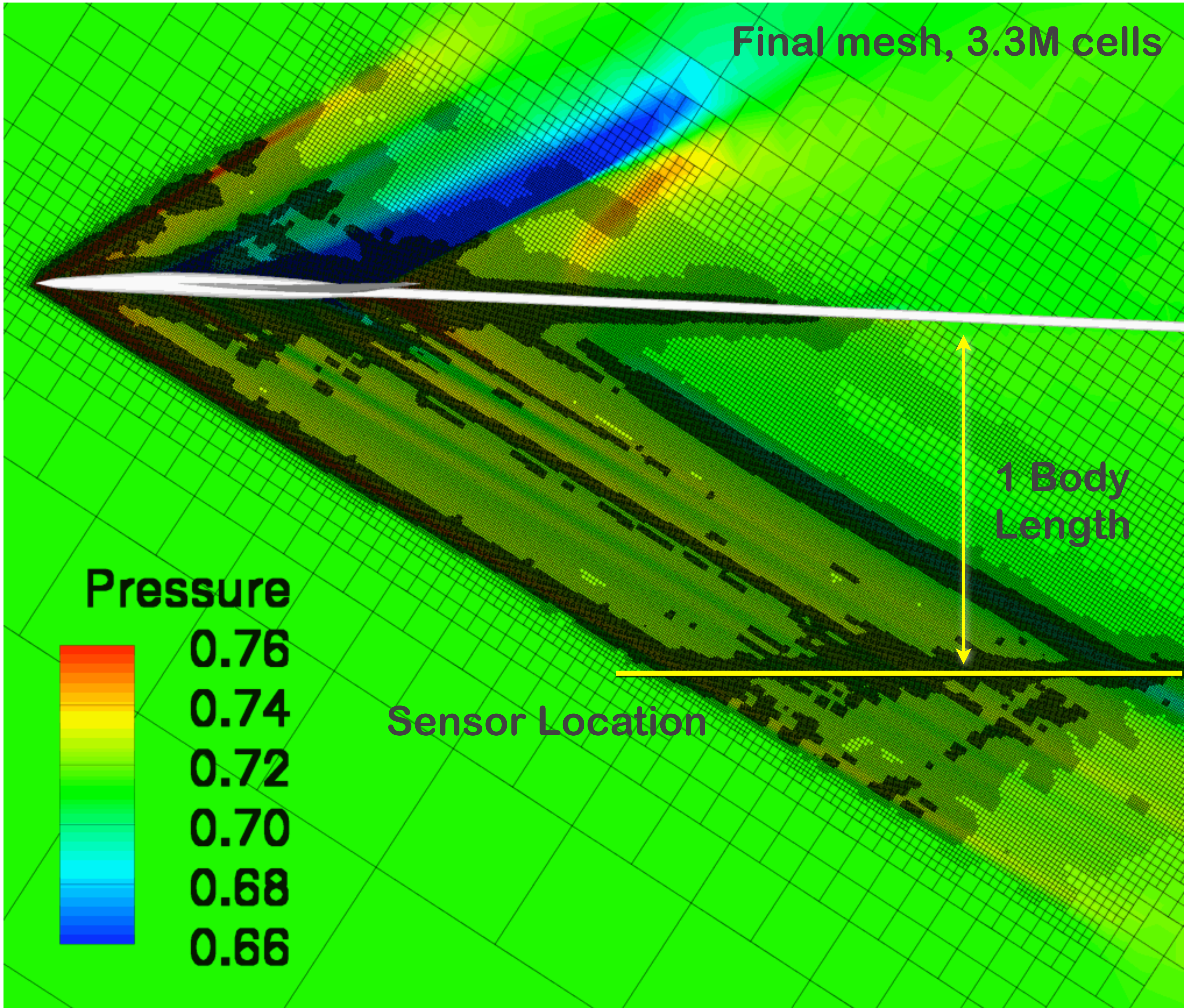
  ★Extensive refinement of volume mesh

  ★Many length-scales and non-linear flow features

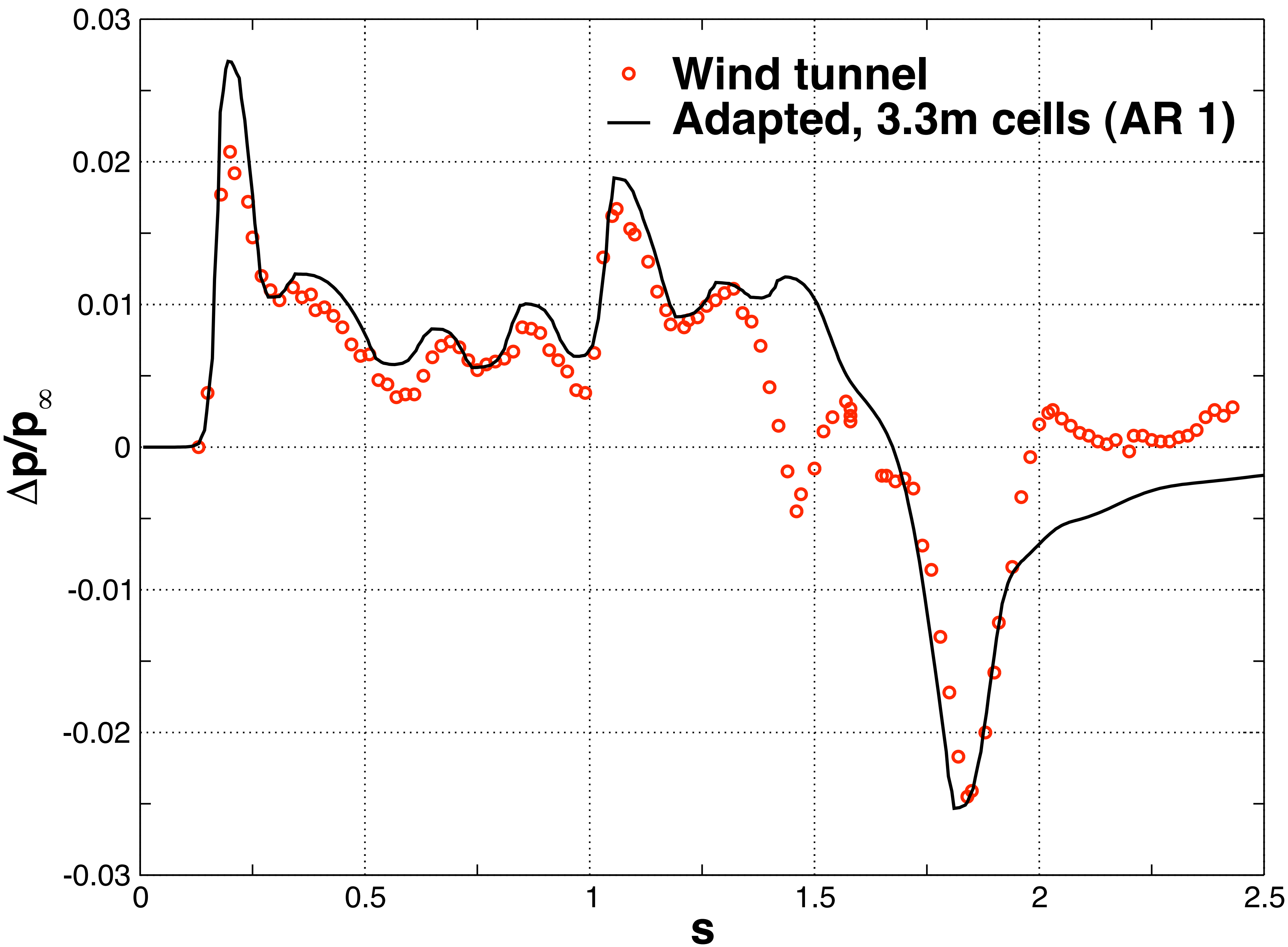


SLSLE Low-Boom Configuration

- $M_\infty = 2.0$, $\alpha = 2.03°$
- Rotate mesh to freestream Mach angle to improve wave propagation to sensor
- Stretch initial mesh along freestream Mach angle (not shown)



SLSLE Pressure Signature



Results
Focus on practical cases

Part A. Classic examples

1. Typical Launch Abort Vehicle database
2. Parametric studies of Launch Vehicles
3. Transport Aircraft
4. Quiet Supersonic Cruise

Part B. Most recent (preliminary) work on cases with jets

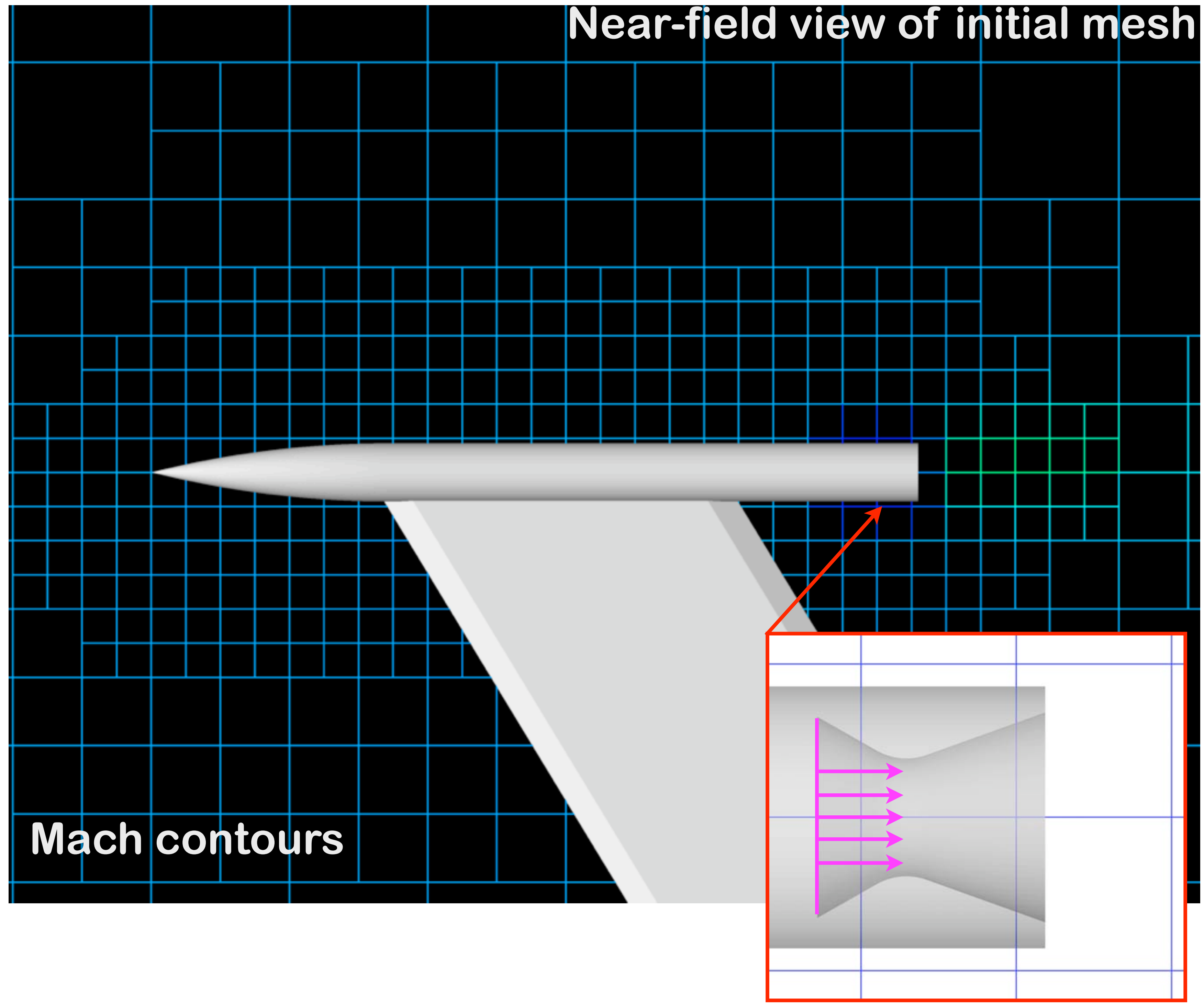
1. Axial Flow Jet
2. Nozzle-Guide-Vane Missile
3. Launch Abort Vehicle with Abort Control Motor Jets

Axial Flow Jet - Problem Setup

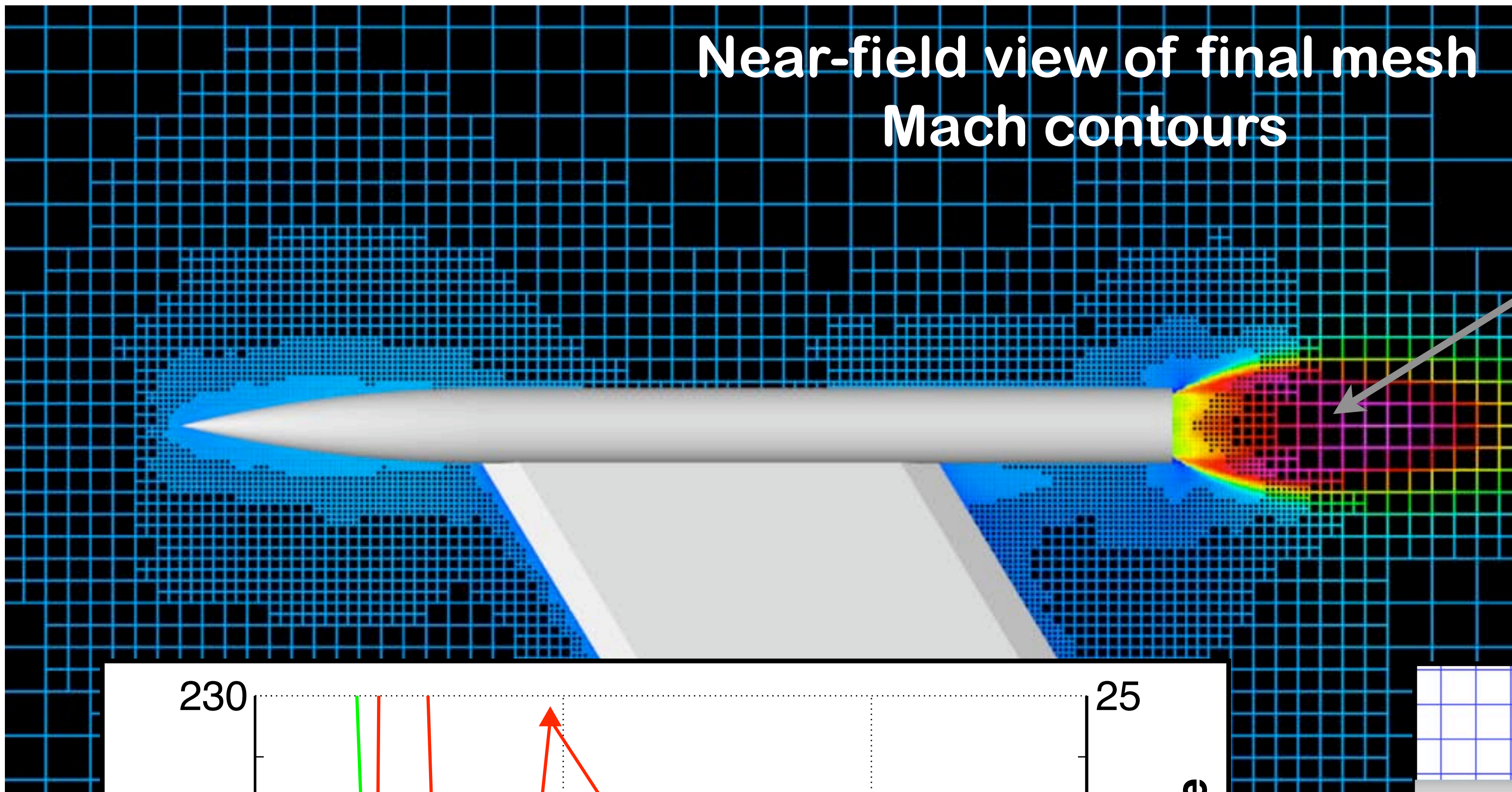
- Initial Mesh: ~17k cells
- Functional: C_A

$$M_\infty = 0.9$$
$$M_{Jet} = 2.7$$
$$P_{Plen.}/P_\infty = 88$$

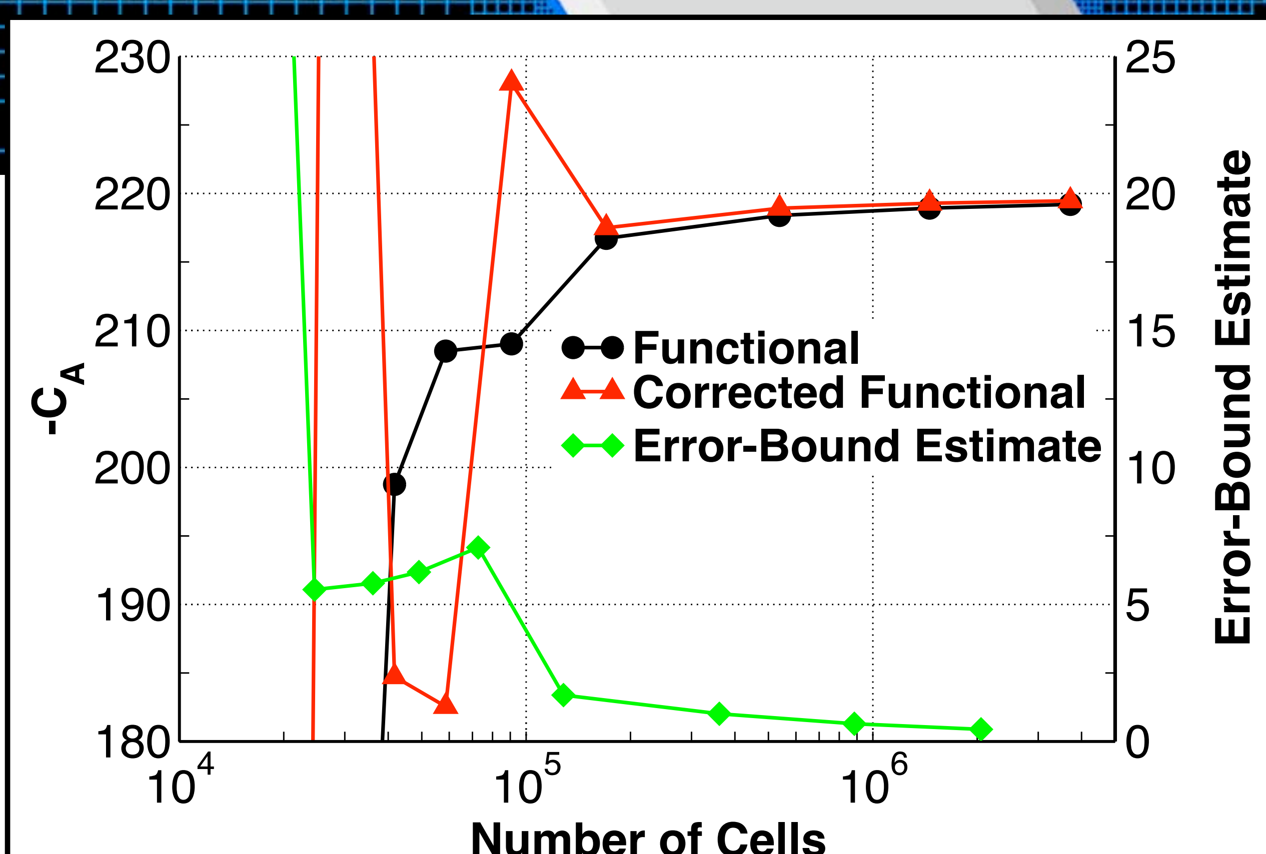
- Power boundary conditions applied at plenum face

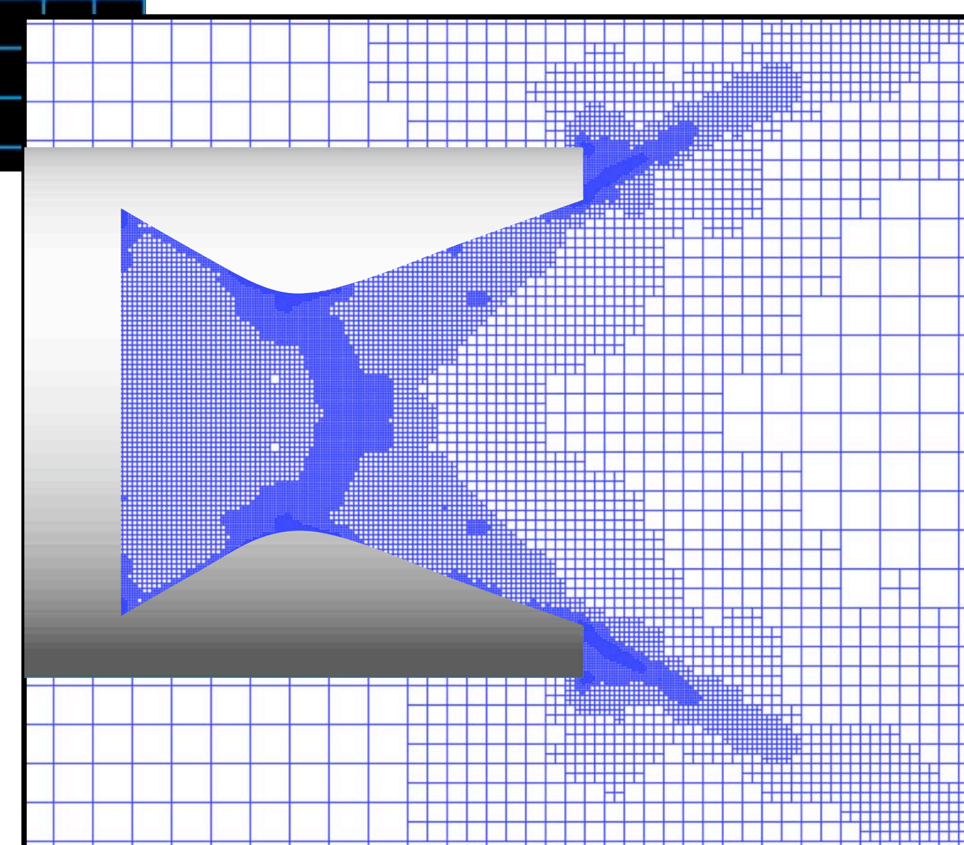


Final Mesh (Functional C_A, TOL=0.5%)



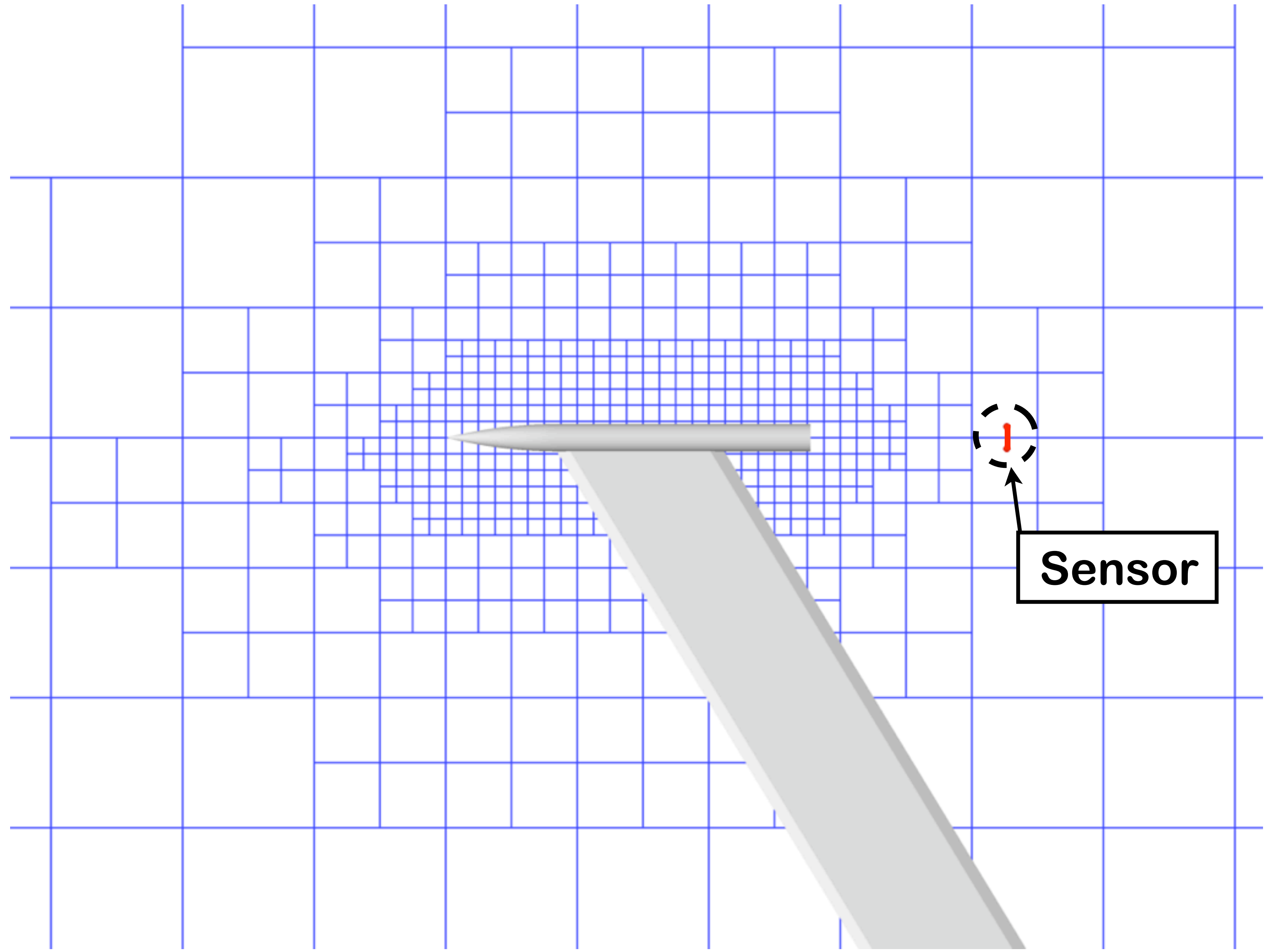
Note rapid coarsening of mesh due to small influence of discretization errors on axial force



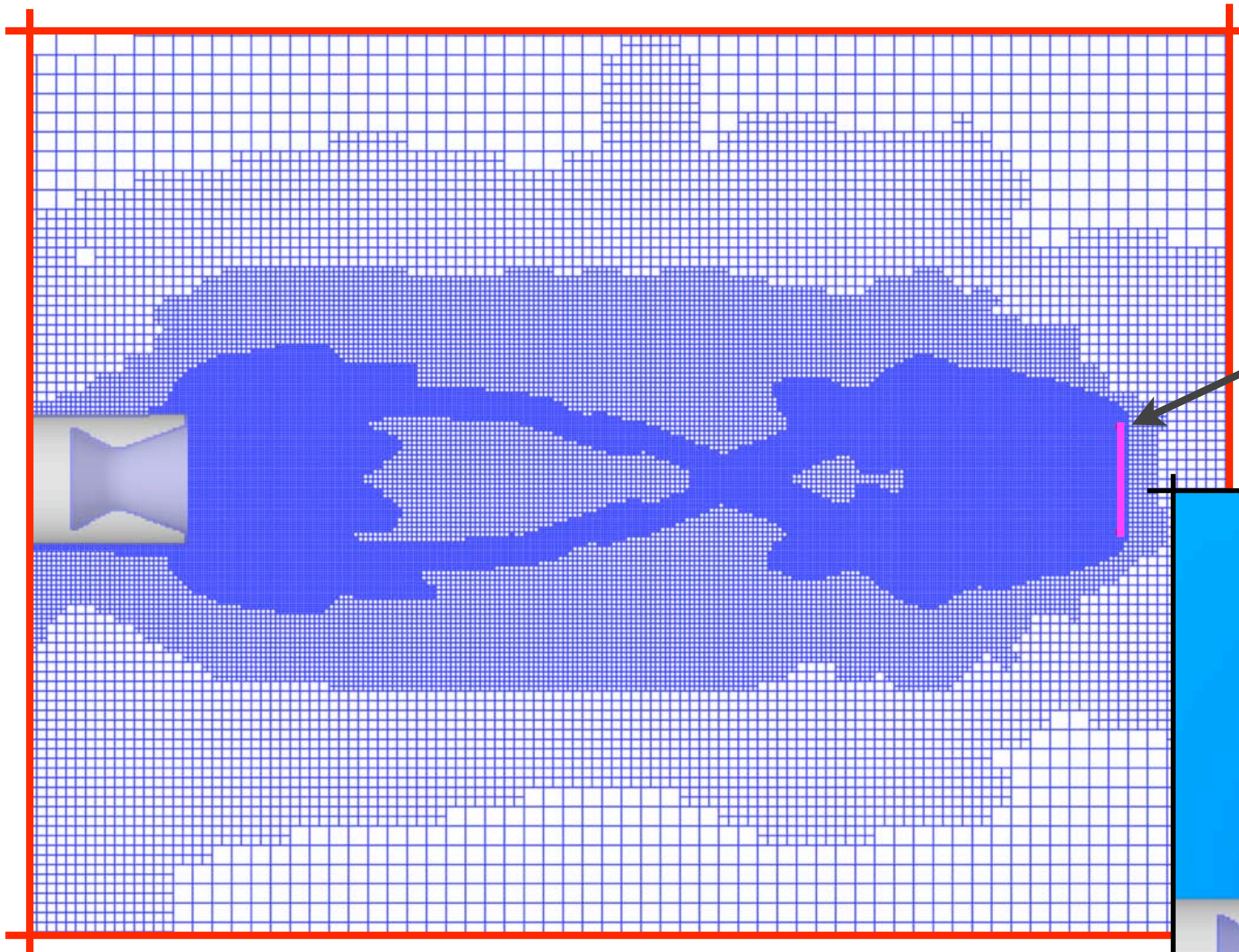


Axial Flow Jet - Field Functional

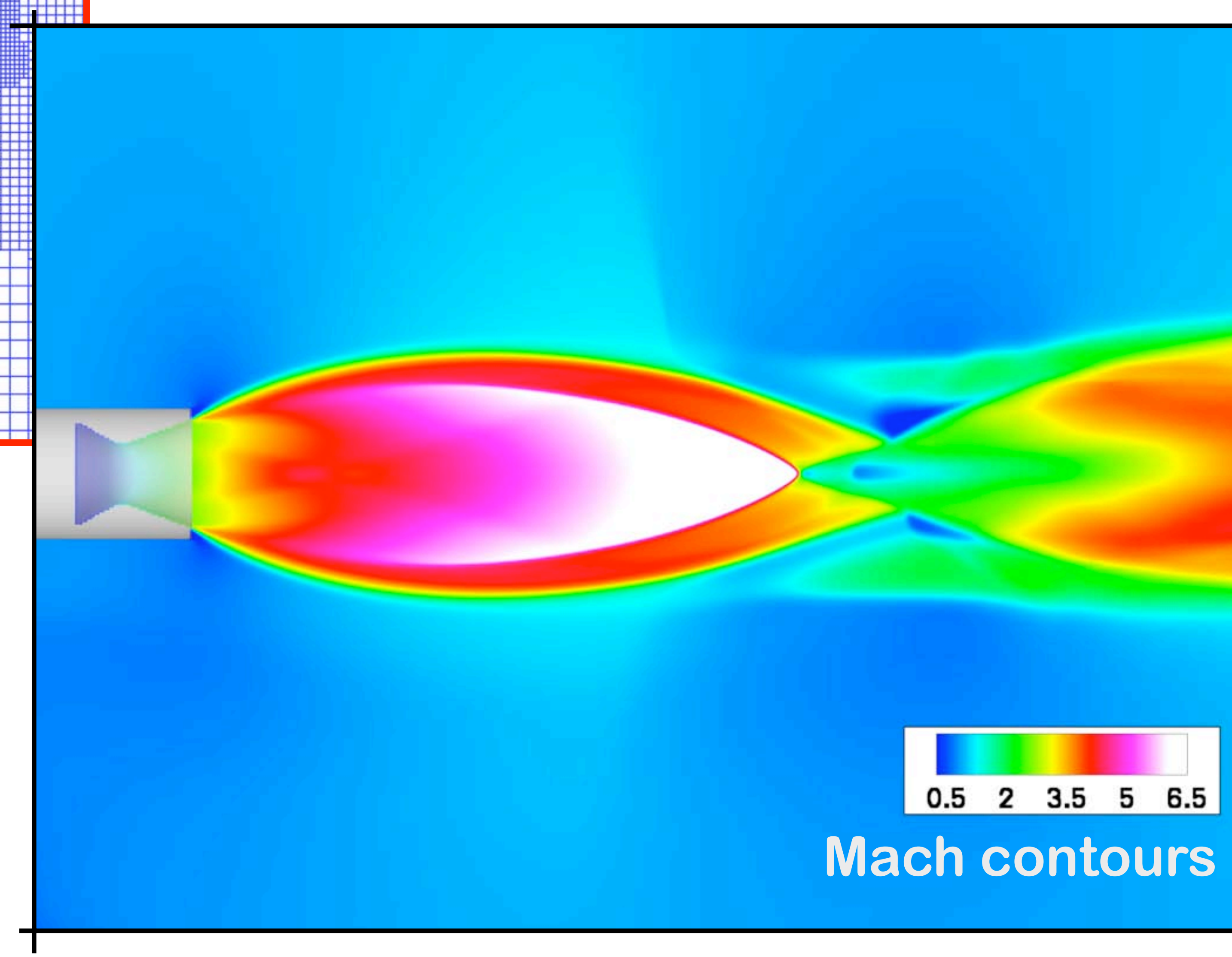
- Example case to demonstrate field functional capability
- Use integral of pressure along a fictitious line sensor to obtain accurate solution in nozzle plume
- Coarse initial mesh (~17k cells)
- Transonic flow conditions



Final Mesh (~9M cells)



$$M_\infty = 0.9$$
$$M_{Jet} = 2.7$$
$$P_{\text{Plen.}}/P_\infty = 88$$



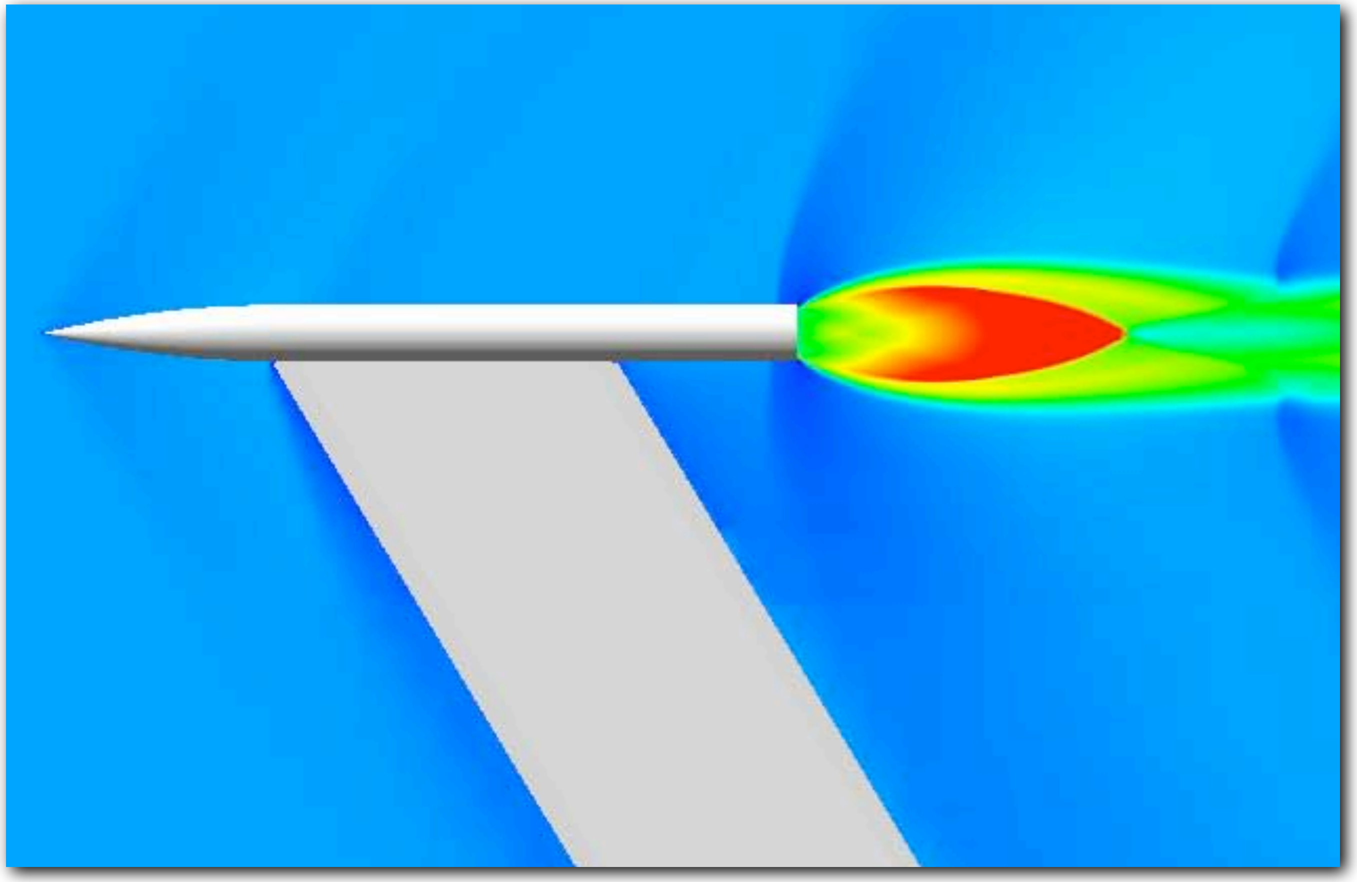
Propulsive Deceleration

- Model problem for propulsive deceleration and control jets

$M_\infty = 2.0$

$M_{Jet} = 2.7$

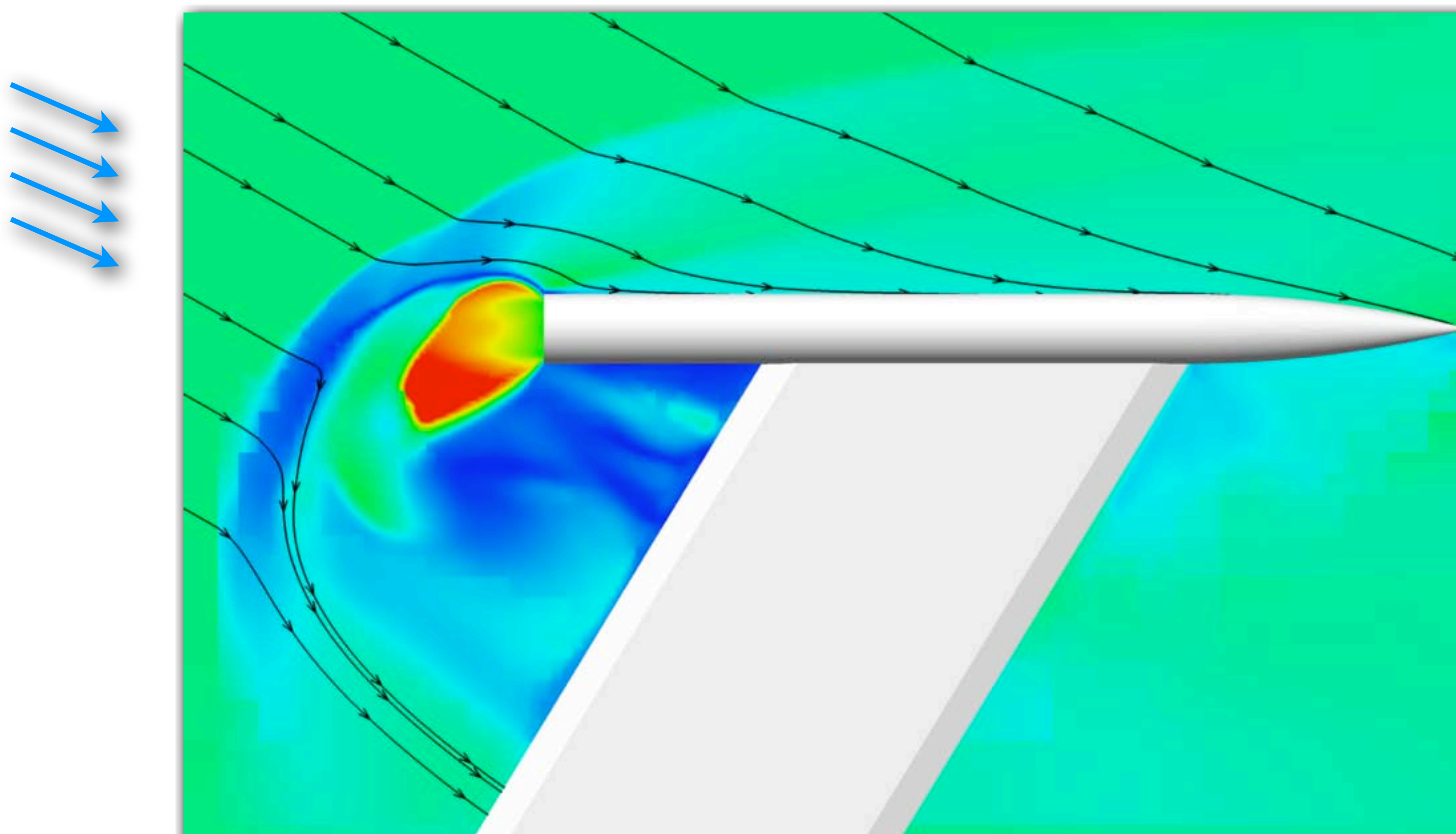
$P_{\mathrm{Plen.}}/P_\infty = 88$



Propulsive Deceleration

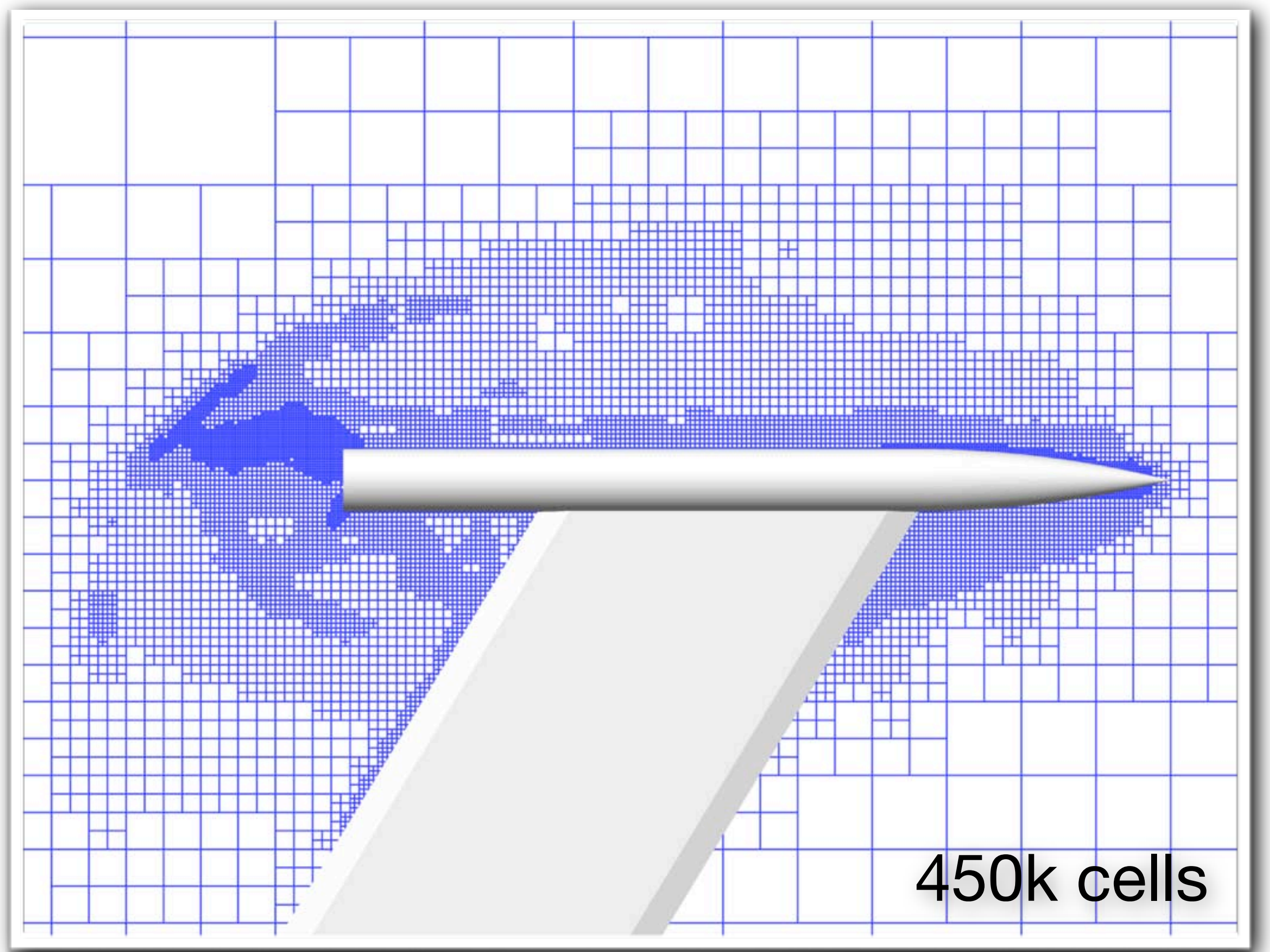
- Model problem for propulsive deceleration and control jets

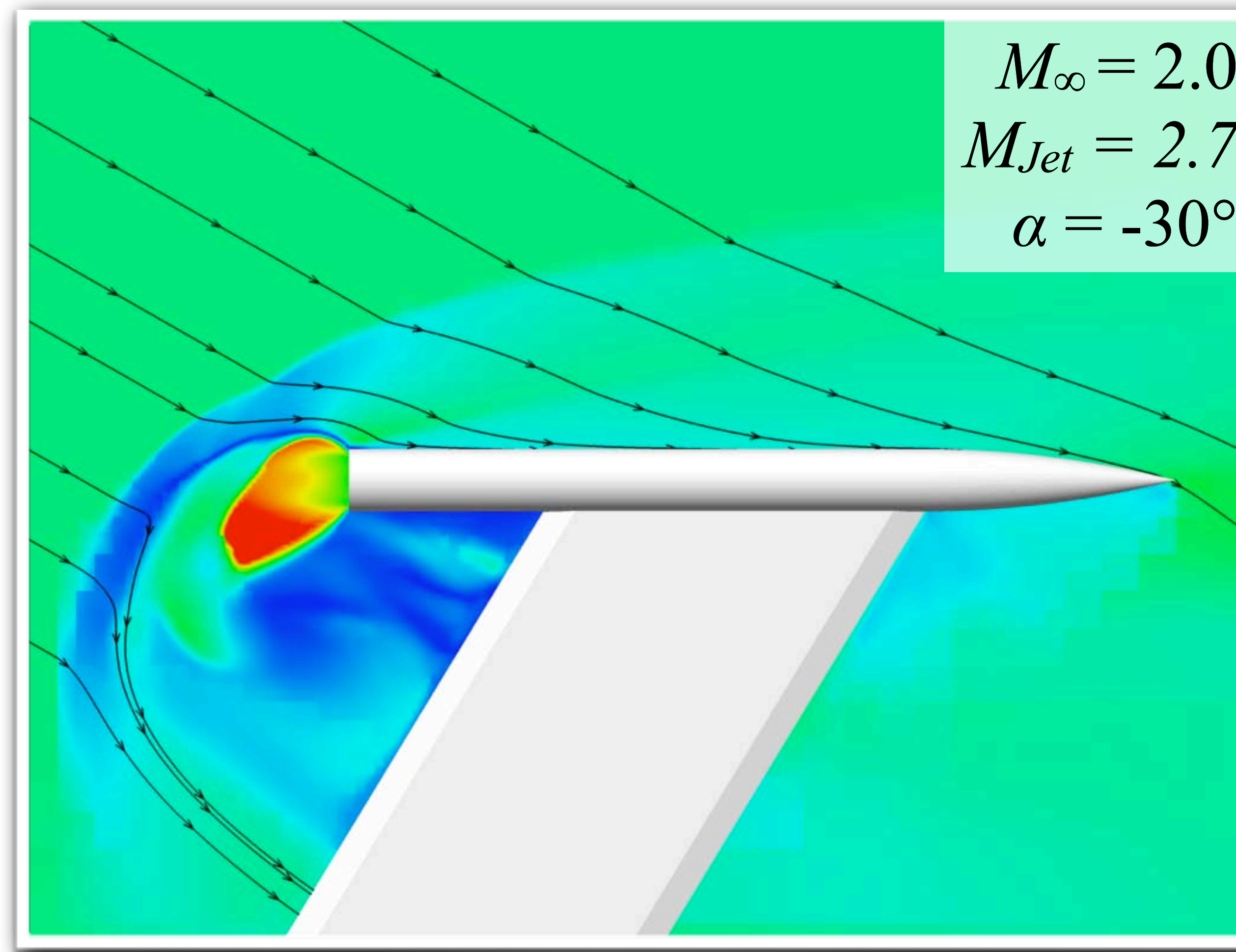
$$M_\infty = 2.0$$
$$M_{Jet} = 2.7$$
$$P_{\text{Plen.}}/P_\infty = 88$$



Propulsive Deceleration

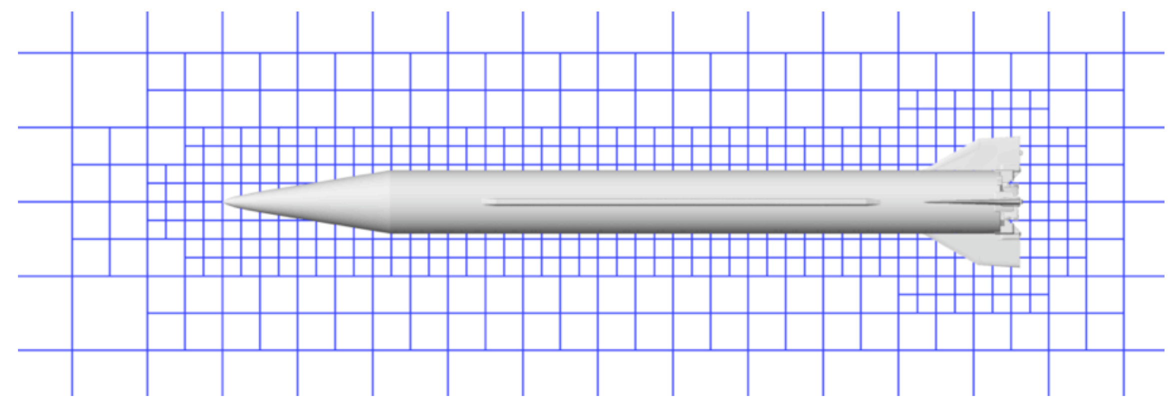
- Model problem for propulsive deceleration and control jets





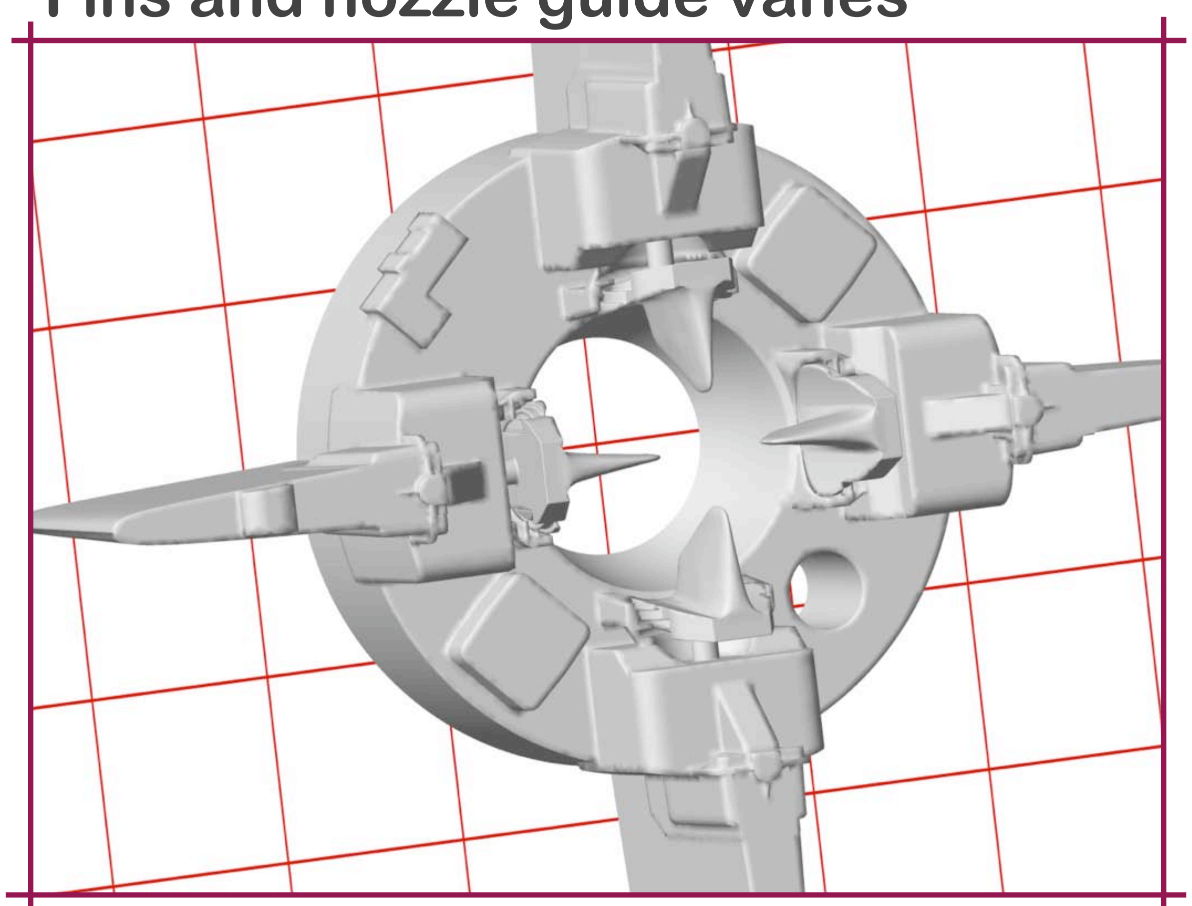
NGV Missile
Functional: Axial force

- Initial mesh: ~5k cells
- Supersonic flow: $M_\infty = 2$, $\alpha = 0°$
- Power boundary conditions applied at plenum face

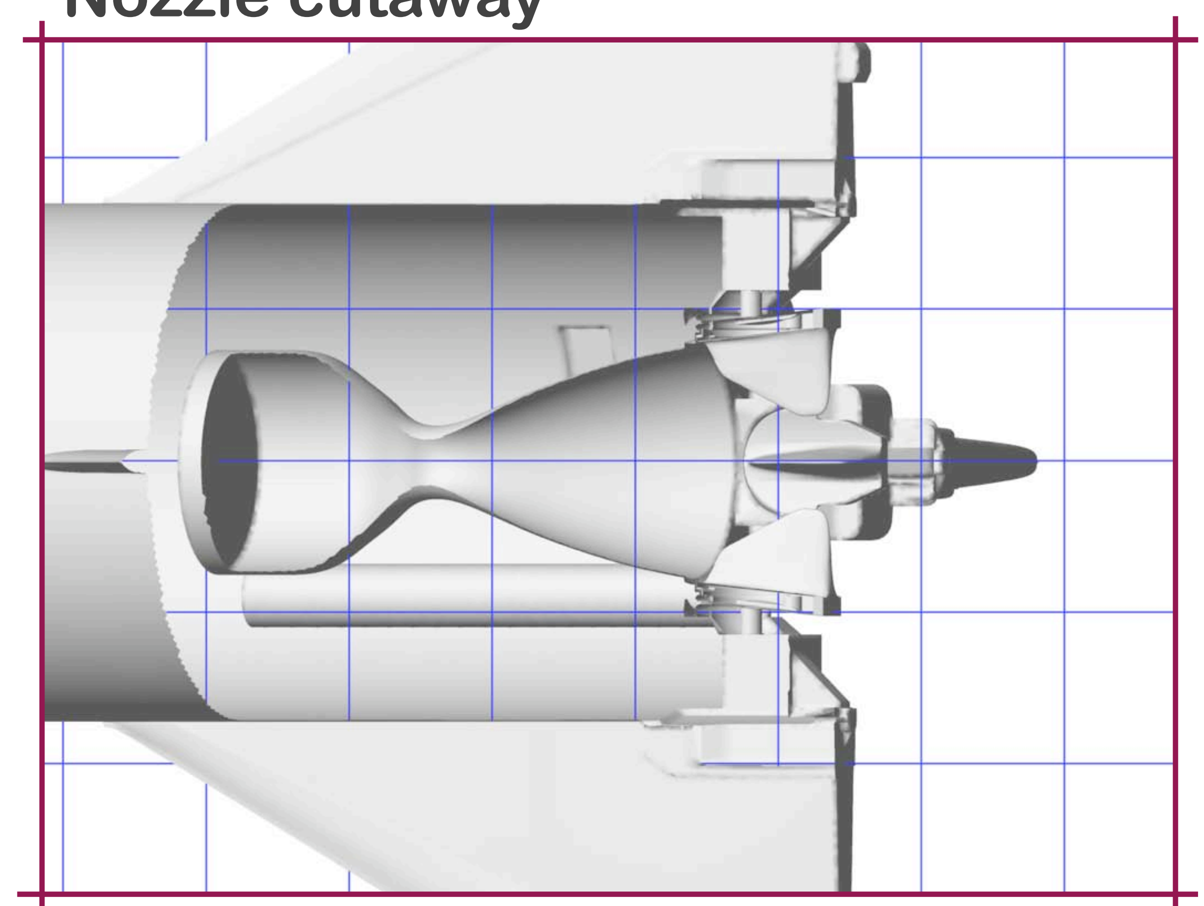


Near-body view of initial mesh

Fins and nozzle guide vanes

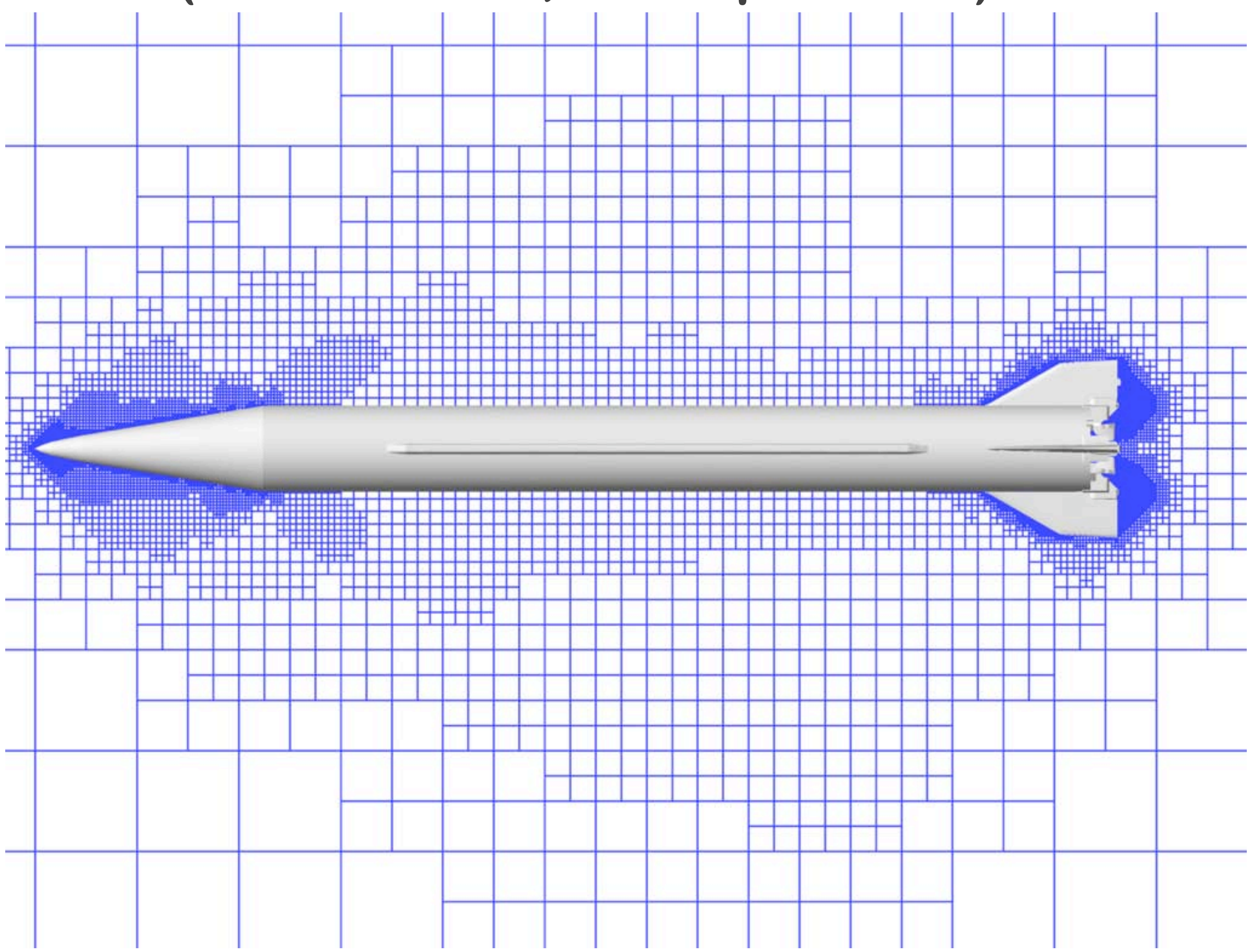


Nozzle cutaway



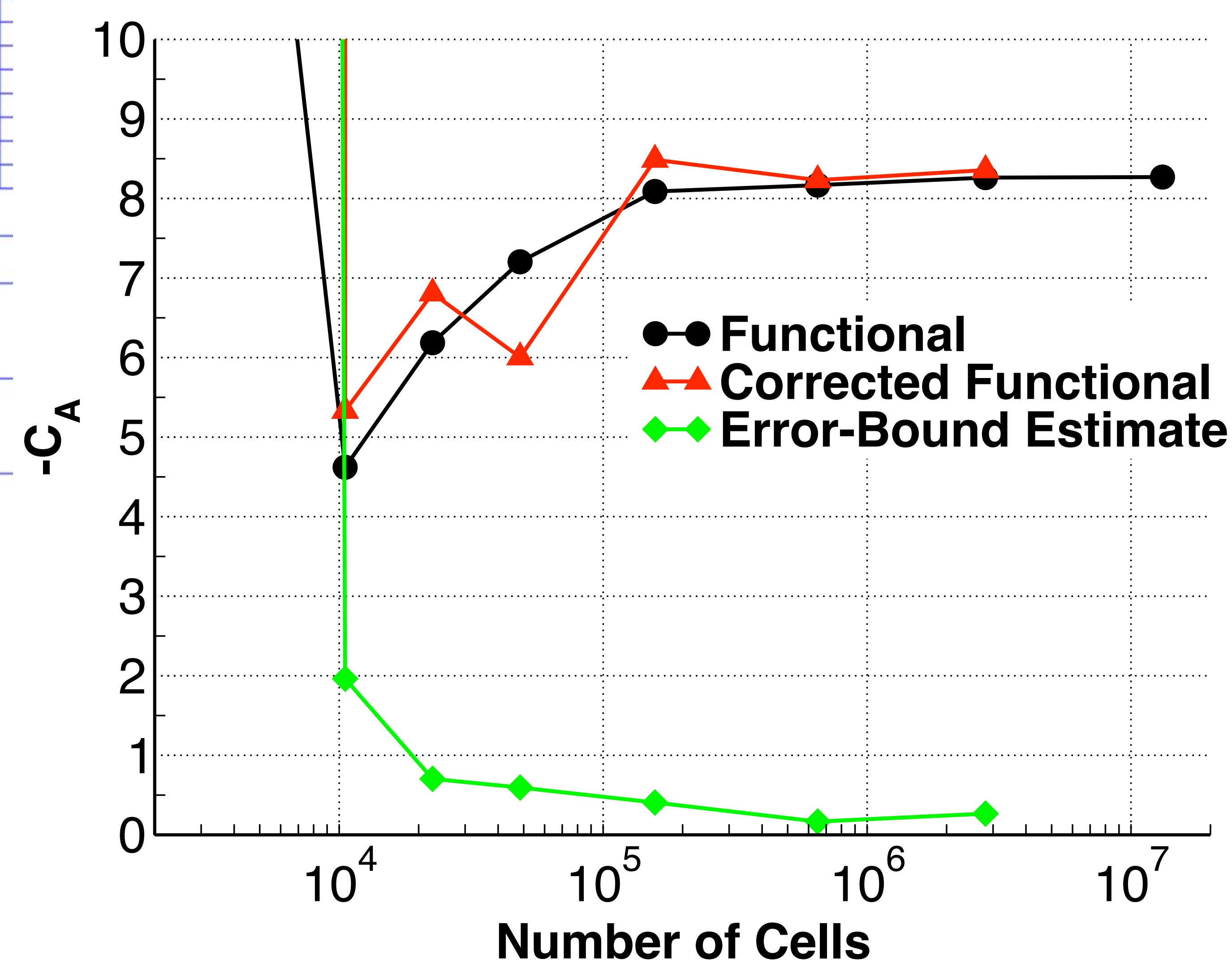
NGV Missile
Functional: Axial force

Near-body view of final mesh
(~ 2.7M cells, 6 adaptations)

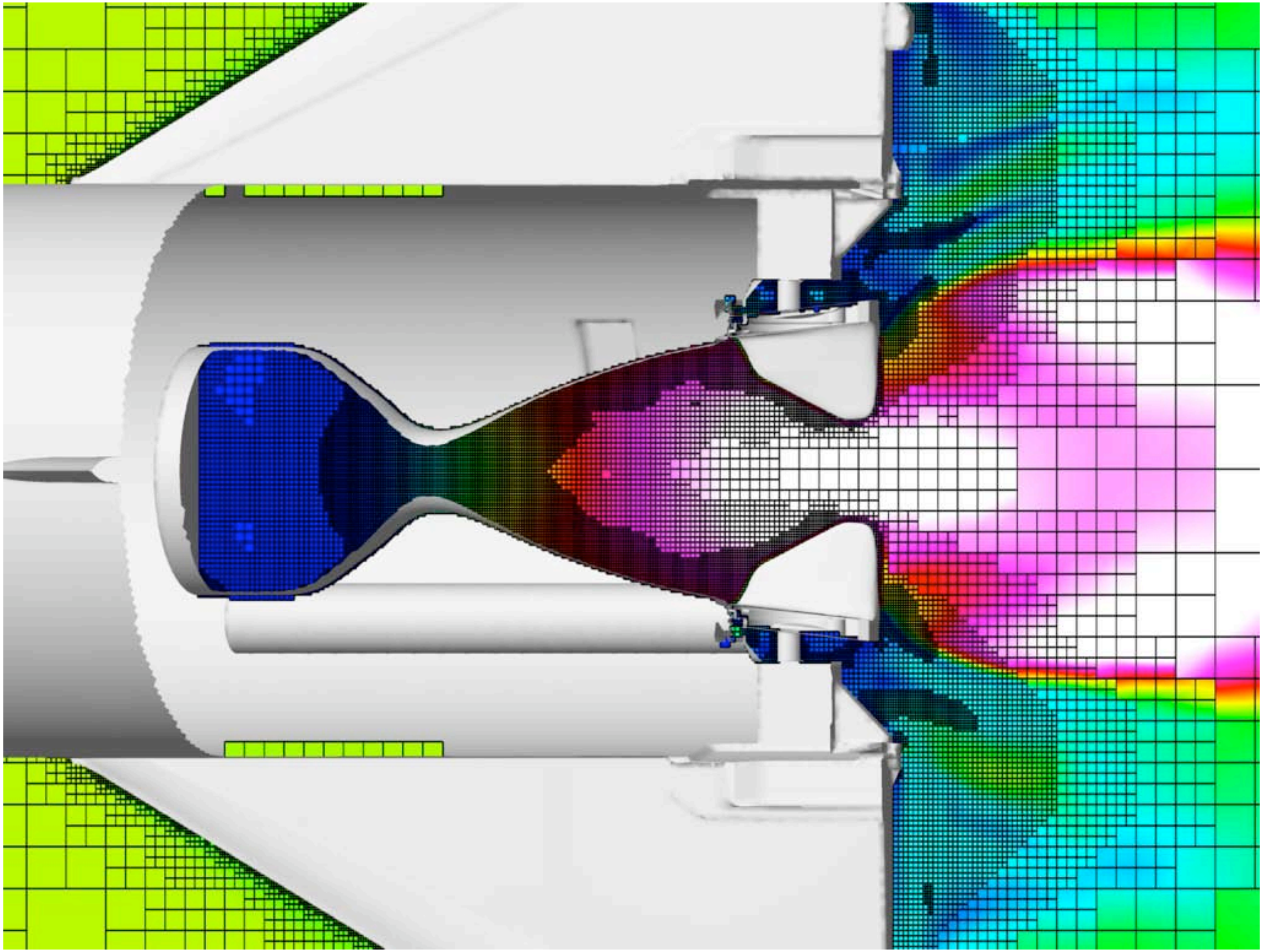


$M_\infty = 2, \alpha = 0°$

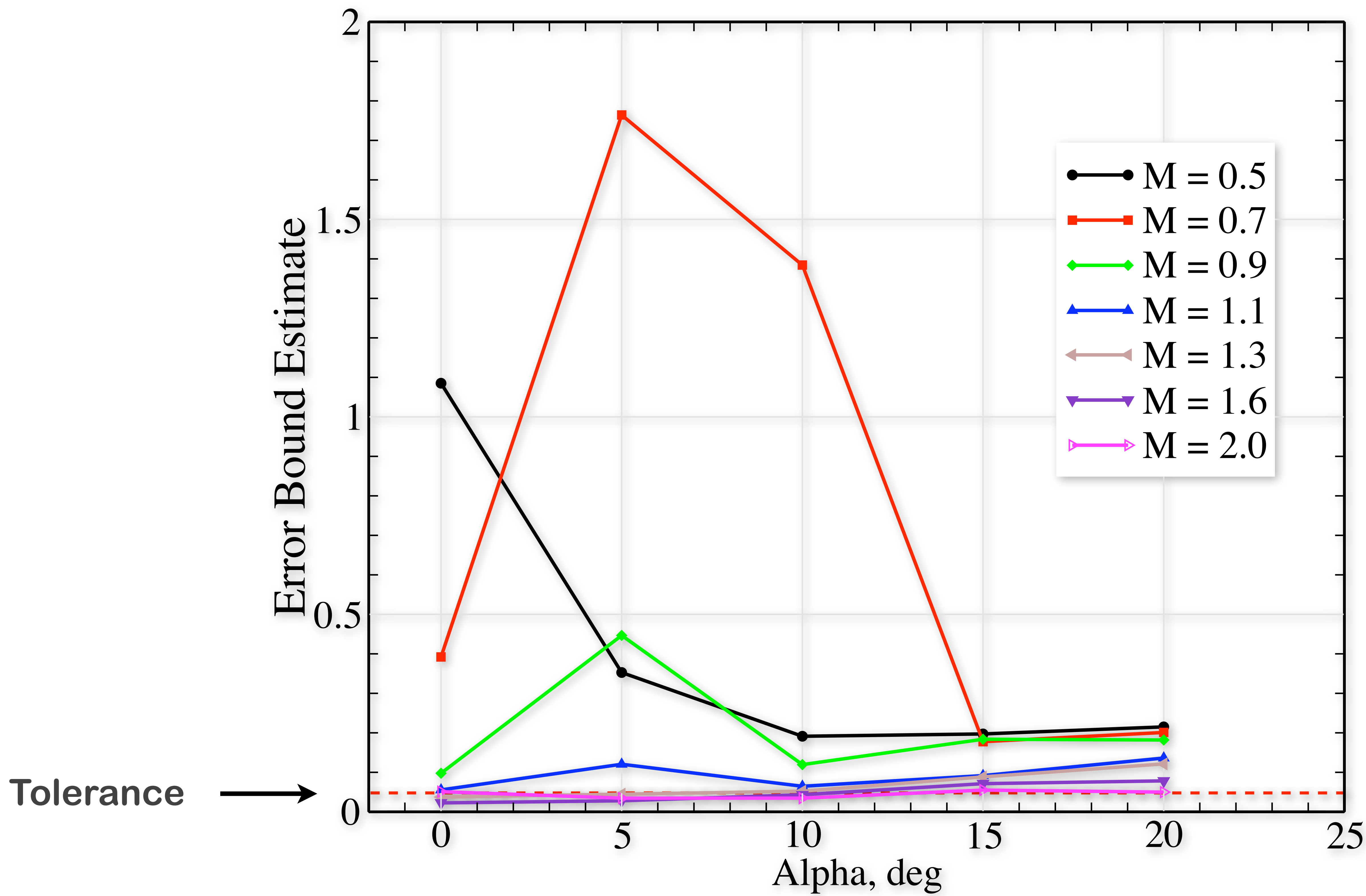
Functional Convergence



NGV Missile
Nozzle cutaway (6 adaptations, Mach contours)



Error Controlled Database



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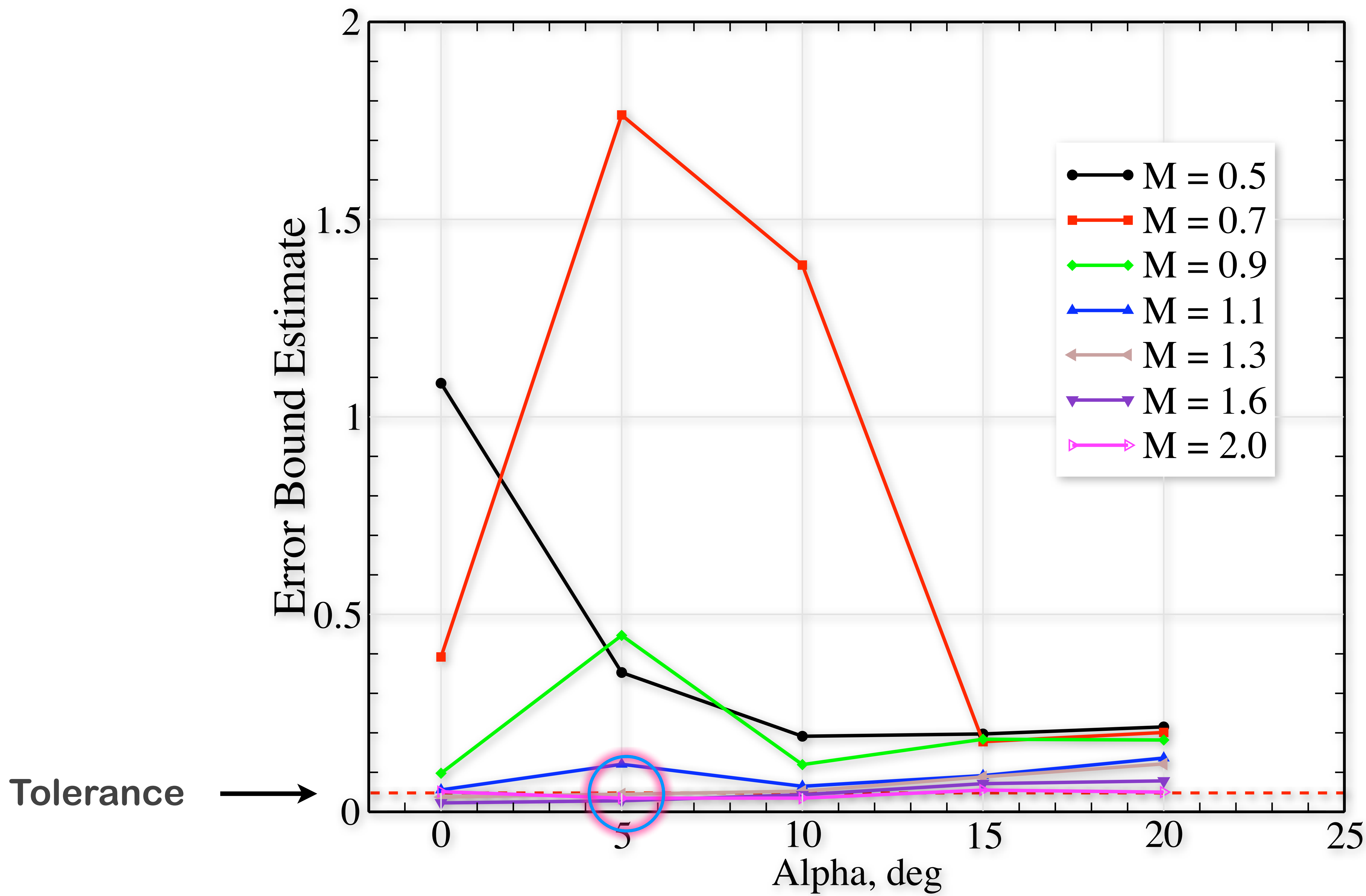

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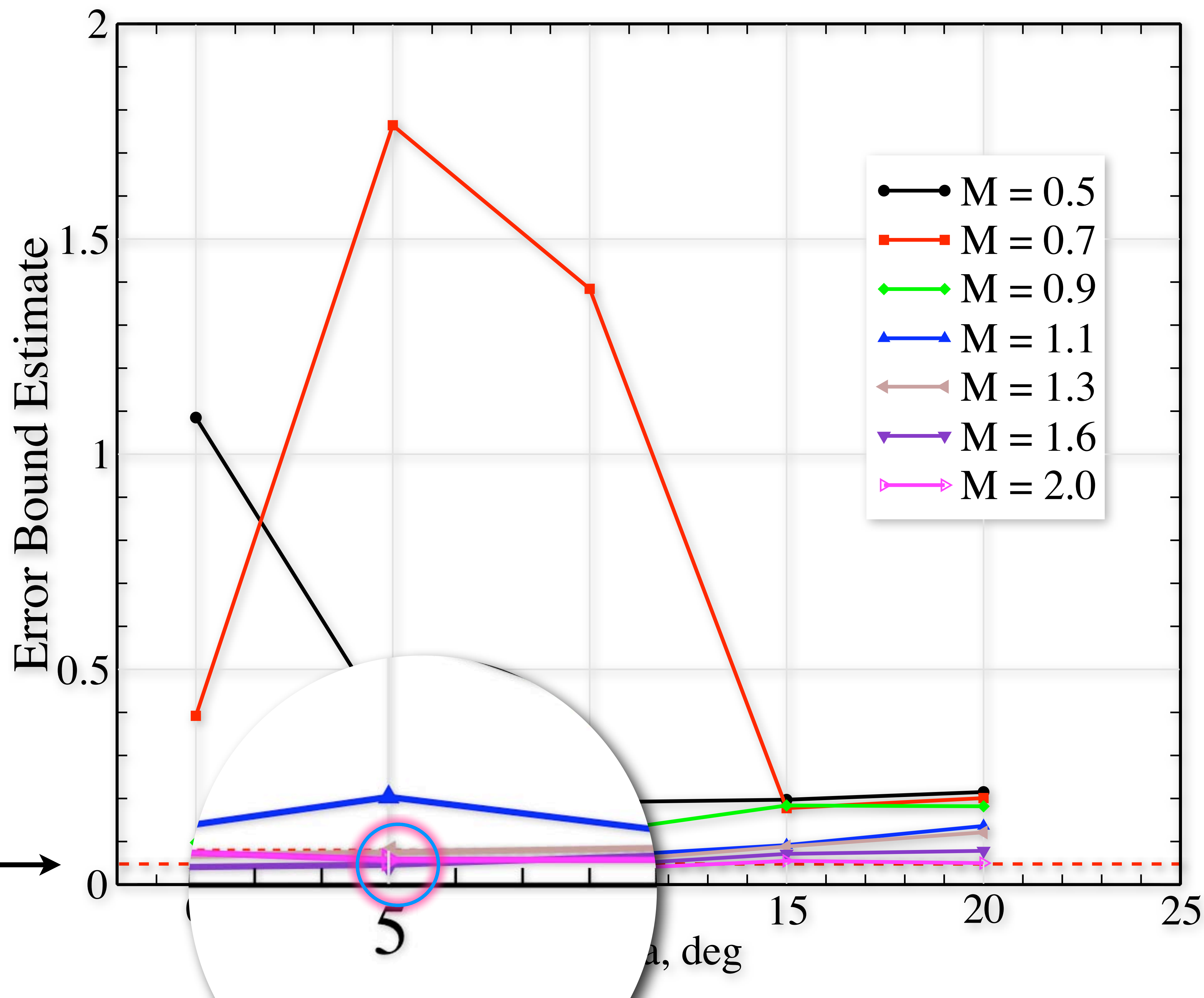
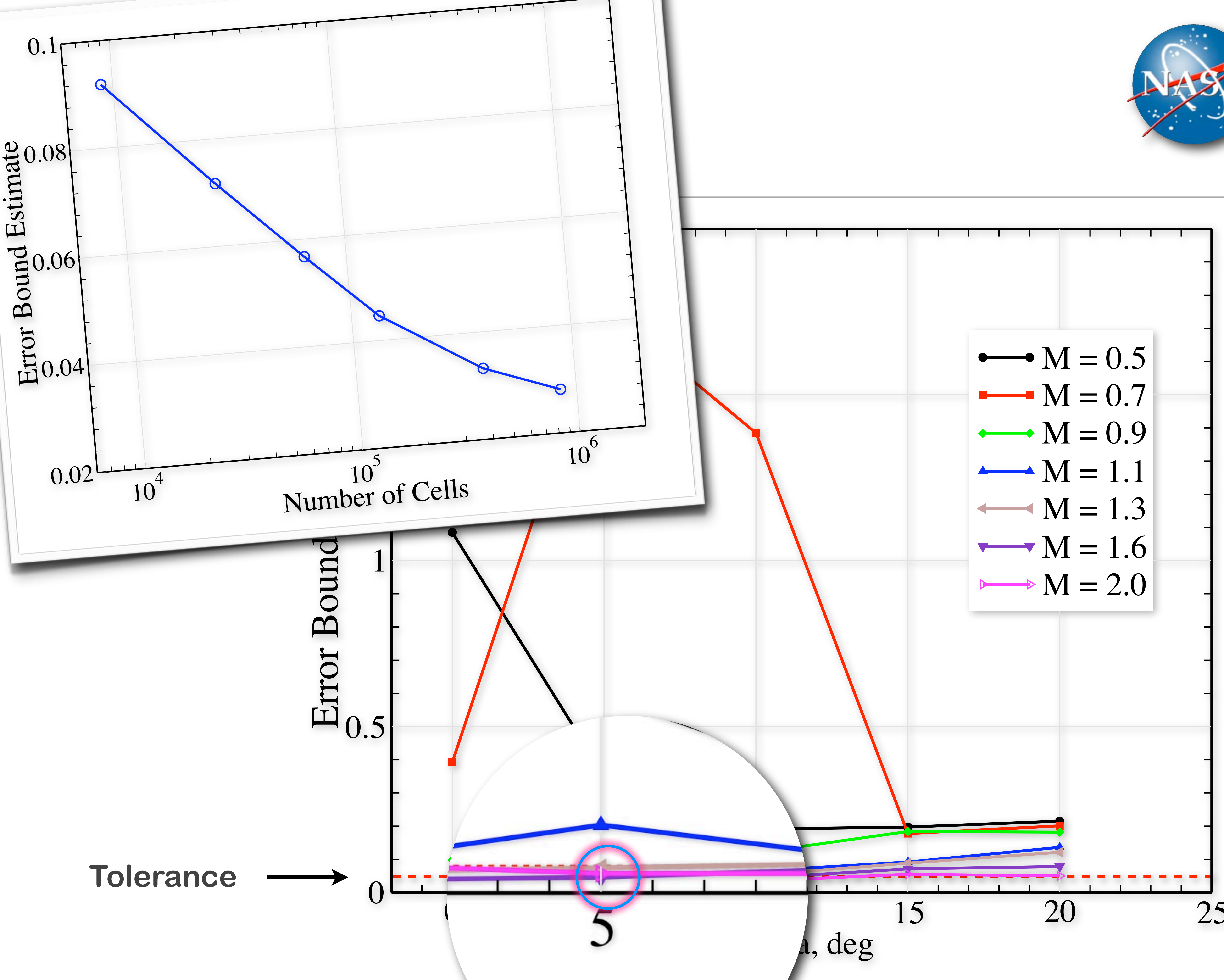
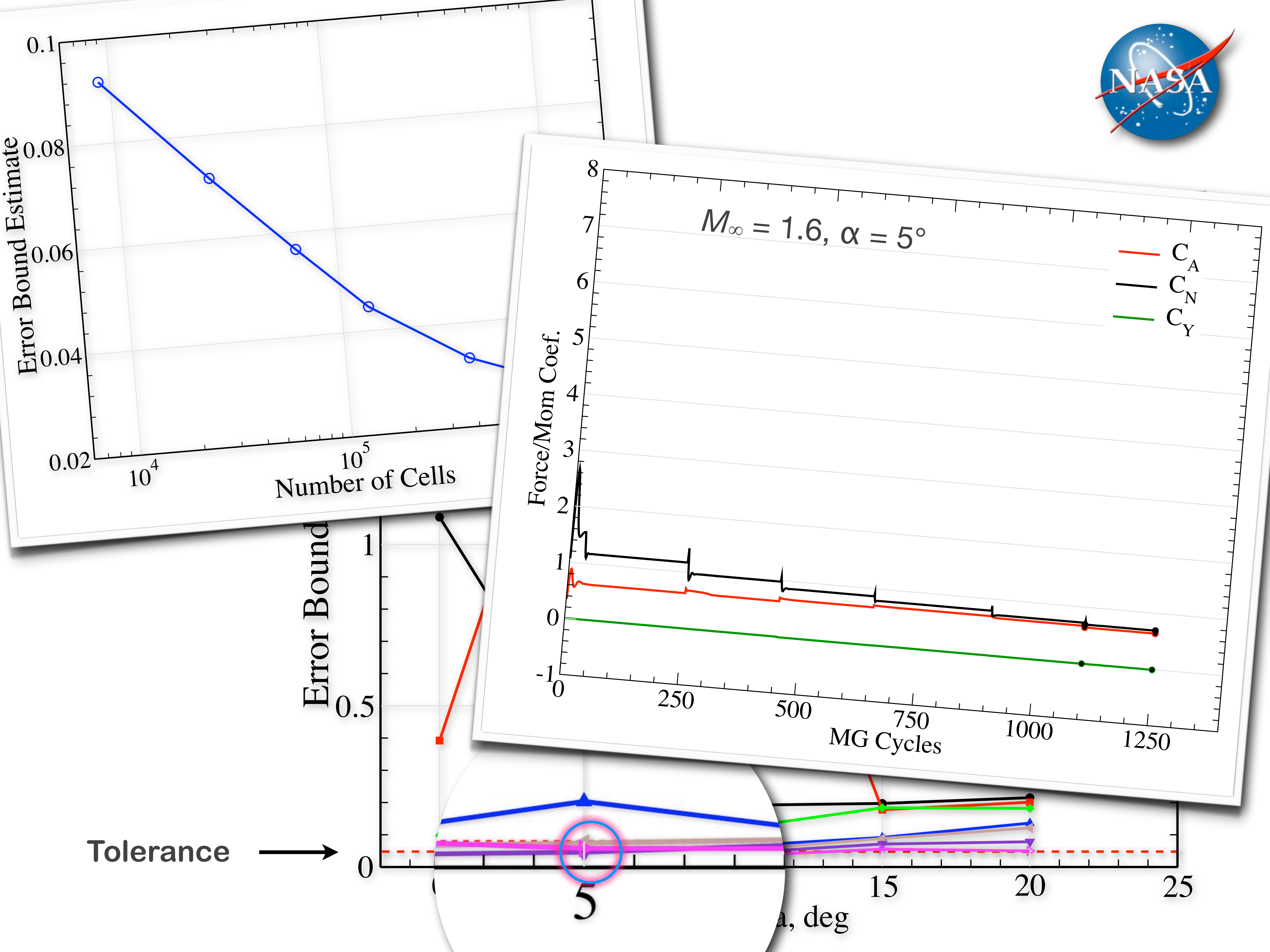


Tolerance

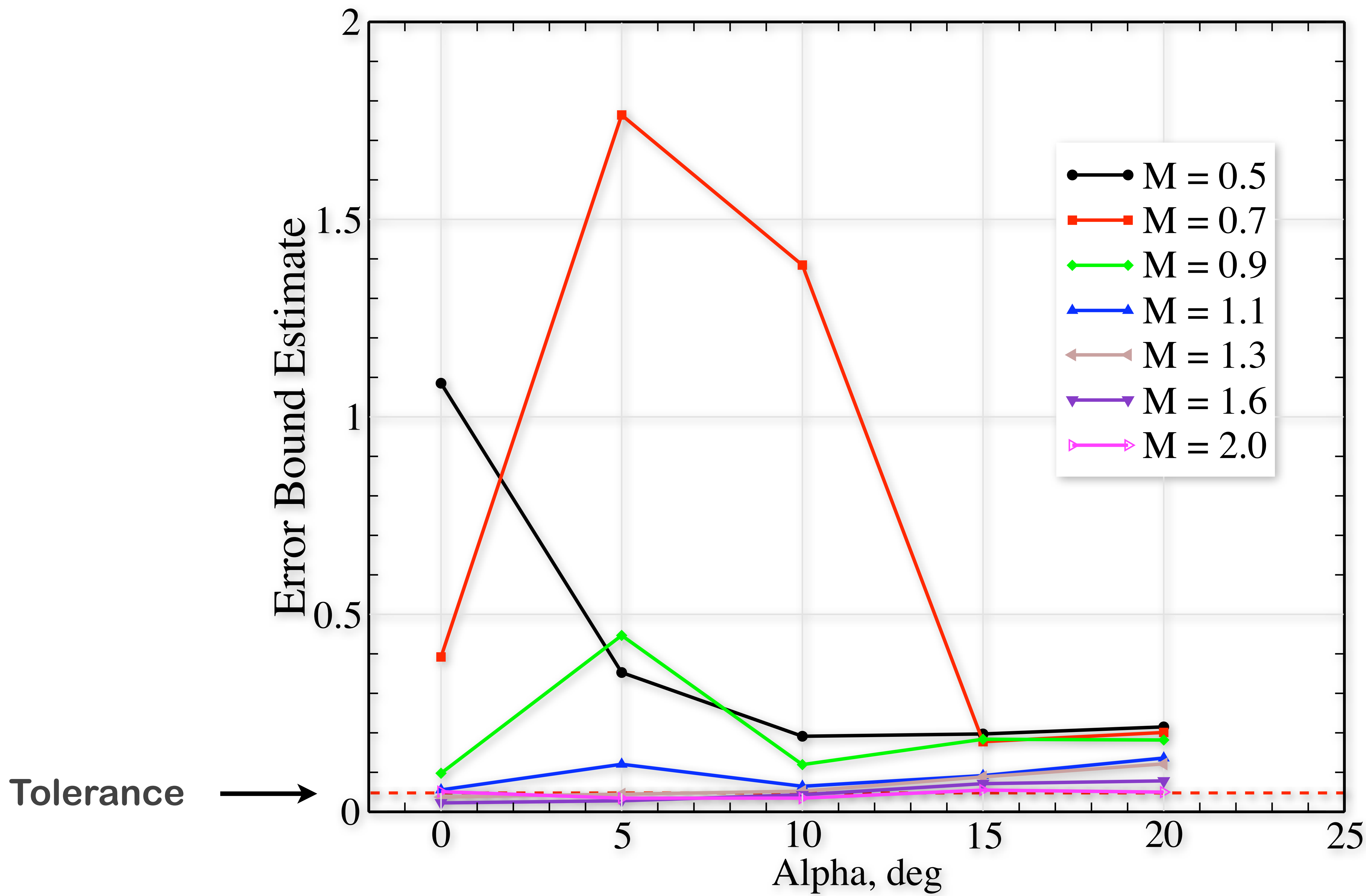


Error Bound Estimate
0.1
0.08
0.06
0.04
0.02
10^4
10^5
10^6
Number of Cells
Error Bound
1
0.5
0
5
15
20
25
a, deg
M = 0.5
M = 0.7
M = 0.9
M = 1.1
M = 1.3
M = 1.6
M = 2.0
Tolerance

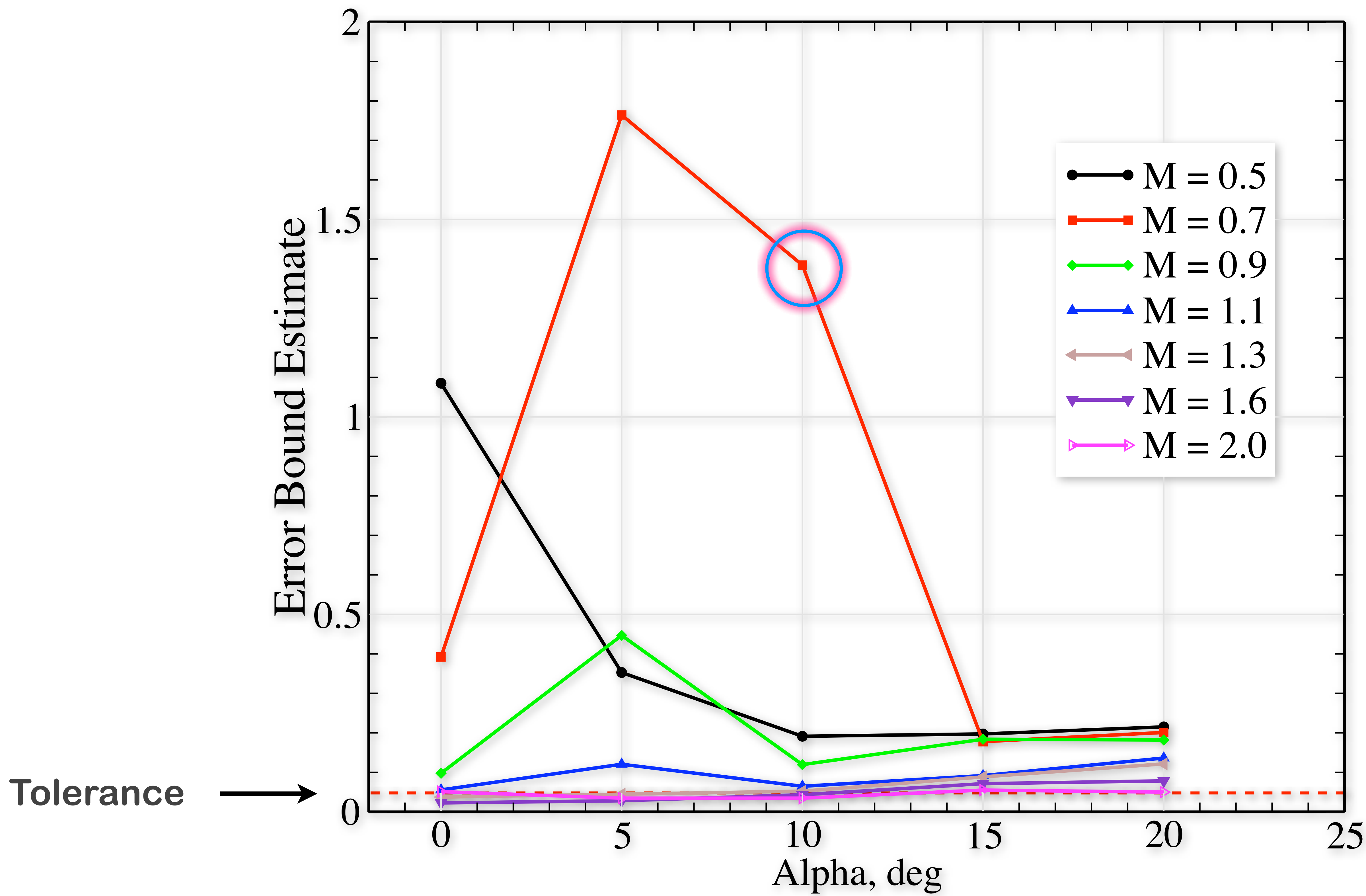


Error Bound Estimate
0.1
0.08
0.06
0.04
0.02
10^4
10^5
Number of Cells
$M_\infty = 1.6,\ \alpha = 5°$
C_A
C_N
C_Y
Force/Mom Coef.
8
7
6
5
4
3
2
1
0
-1
0
250
500
750
1000
1250
MG Cycles
Error Bound
1
0.5
0
Tolerance
5
15
20
25
a, deg

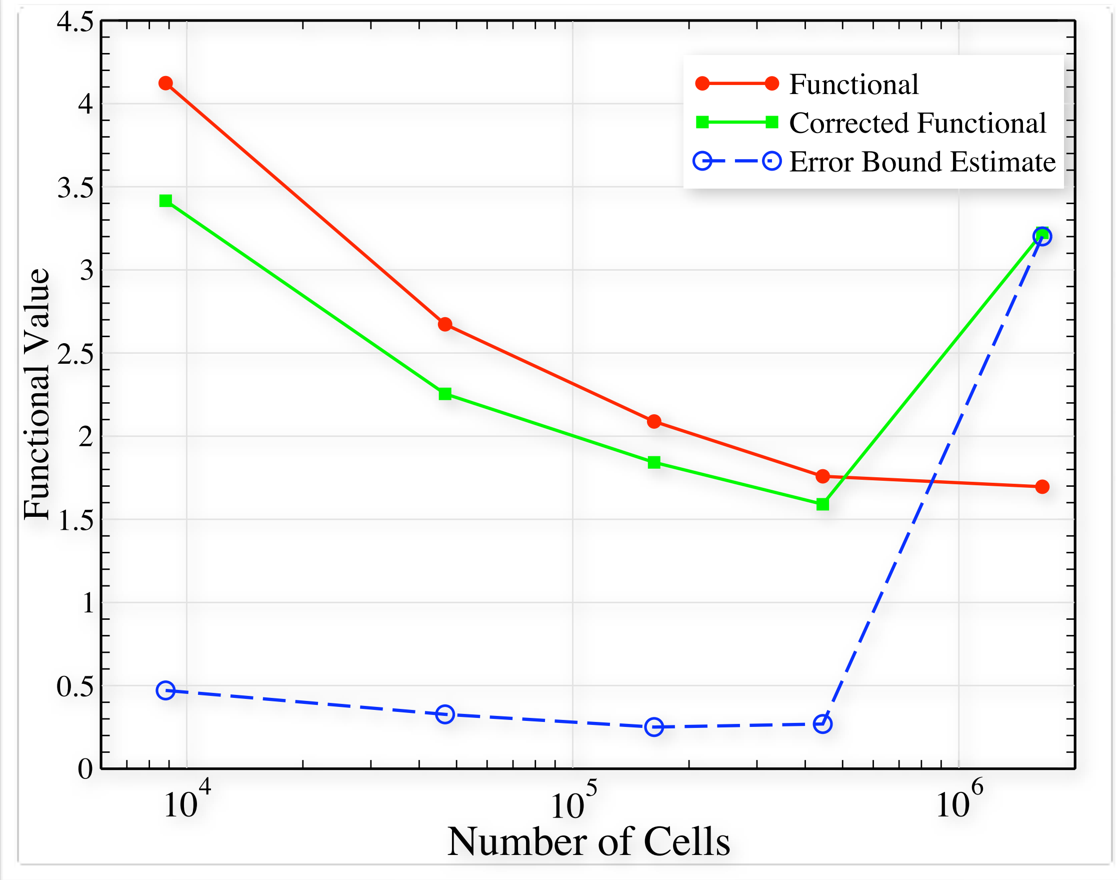
Error Controlled Database

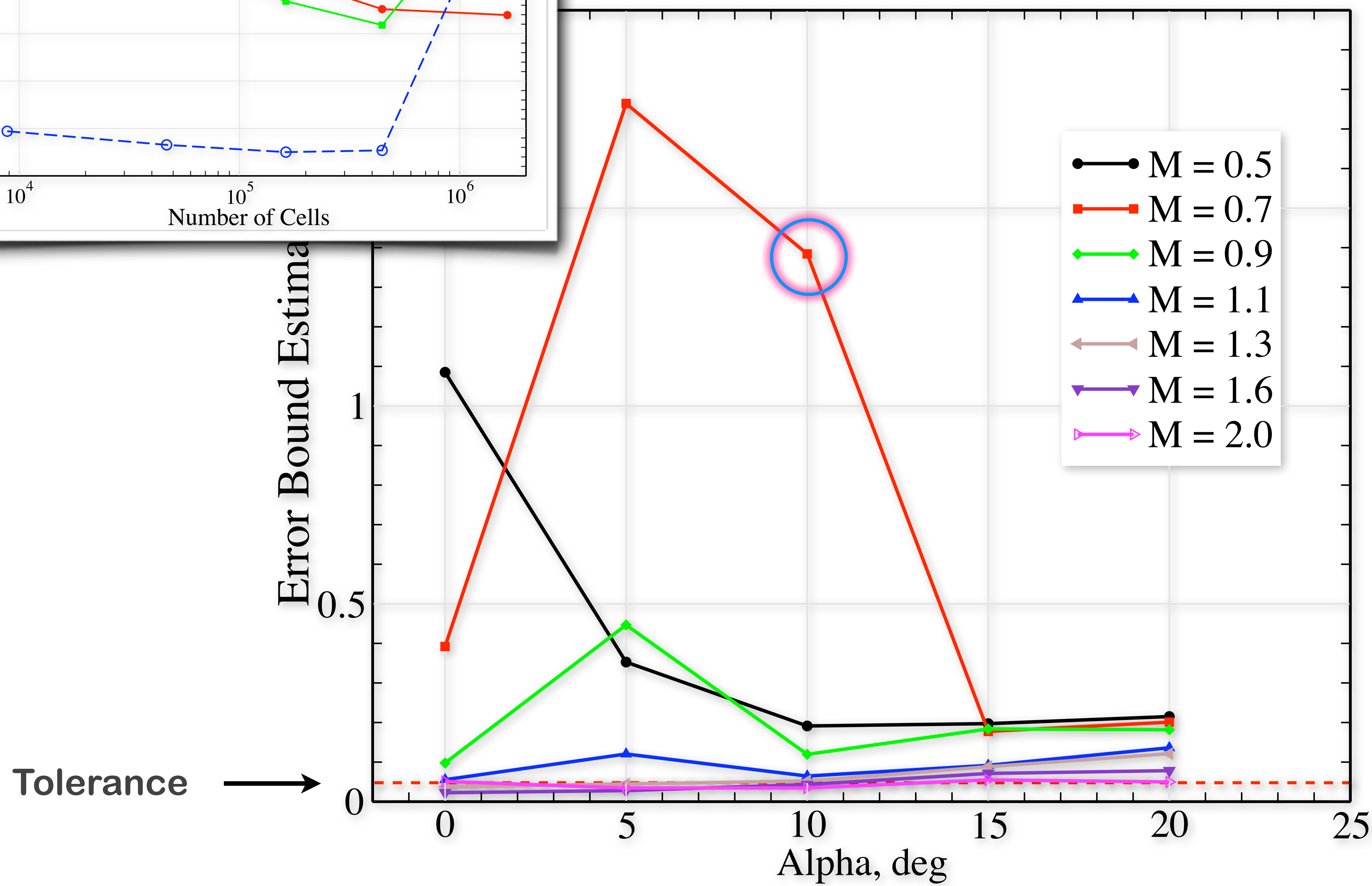


Error Controlled Database

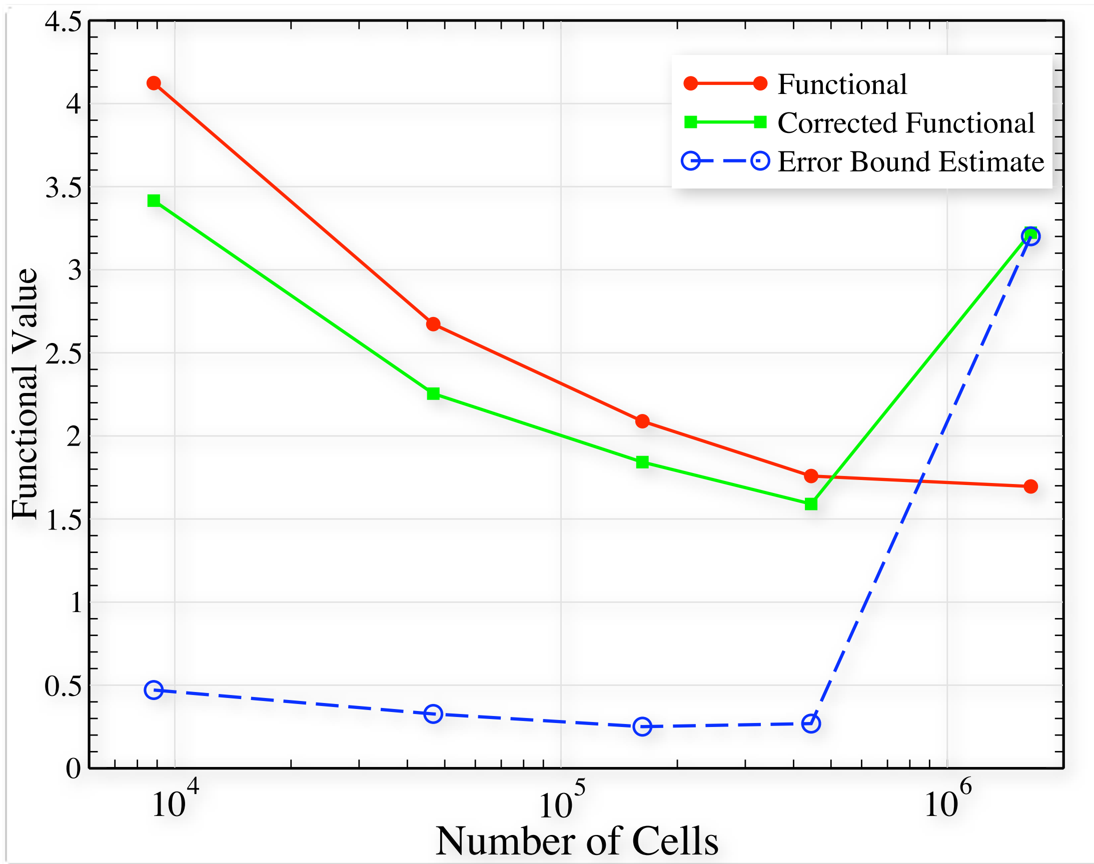


base

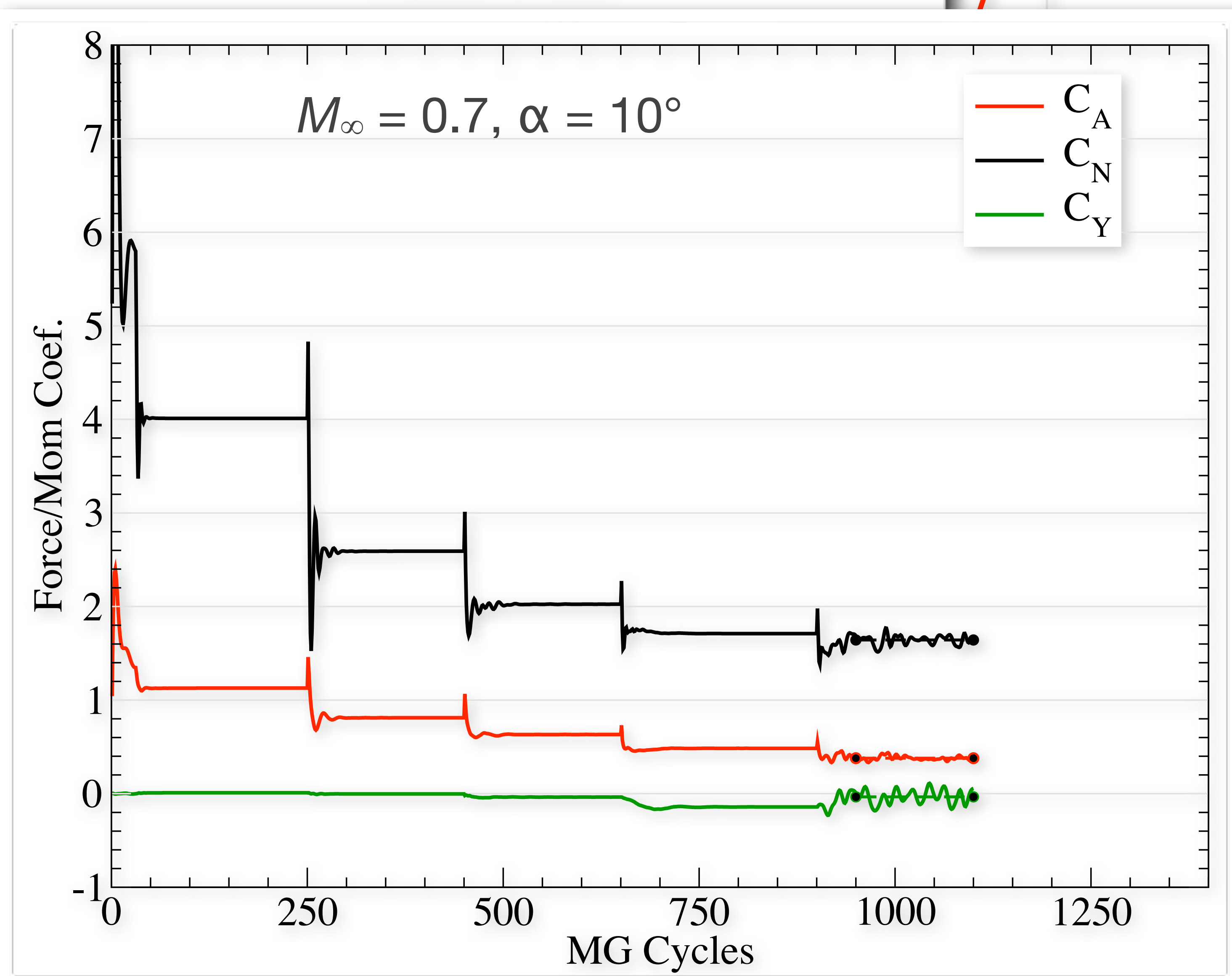

Functional
Corrected Functional
Error Bound Estimate
Functional Value
Number of Cells
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
10^4
10^5
10^6


M = 0.5
M = 0.7
M = 0.9
M = 1.1
M = 1.3
M = 1.6
M = 2.0
Error Bound Estima
Alpha, deg
Tolerance
0
0.5
1
0
5
10
15
20
25

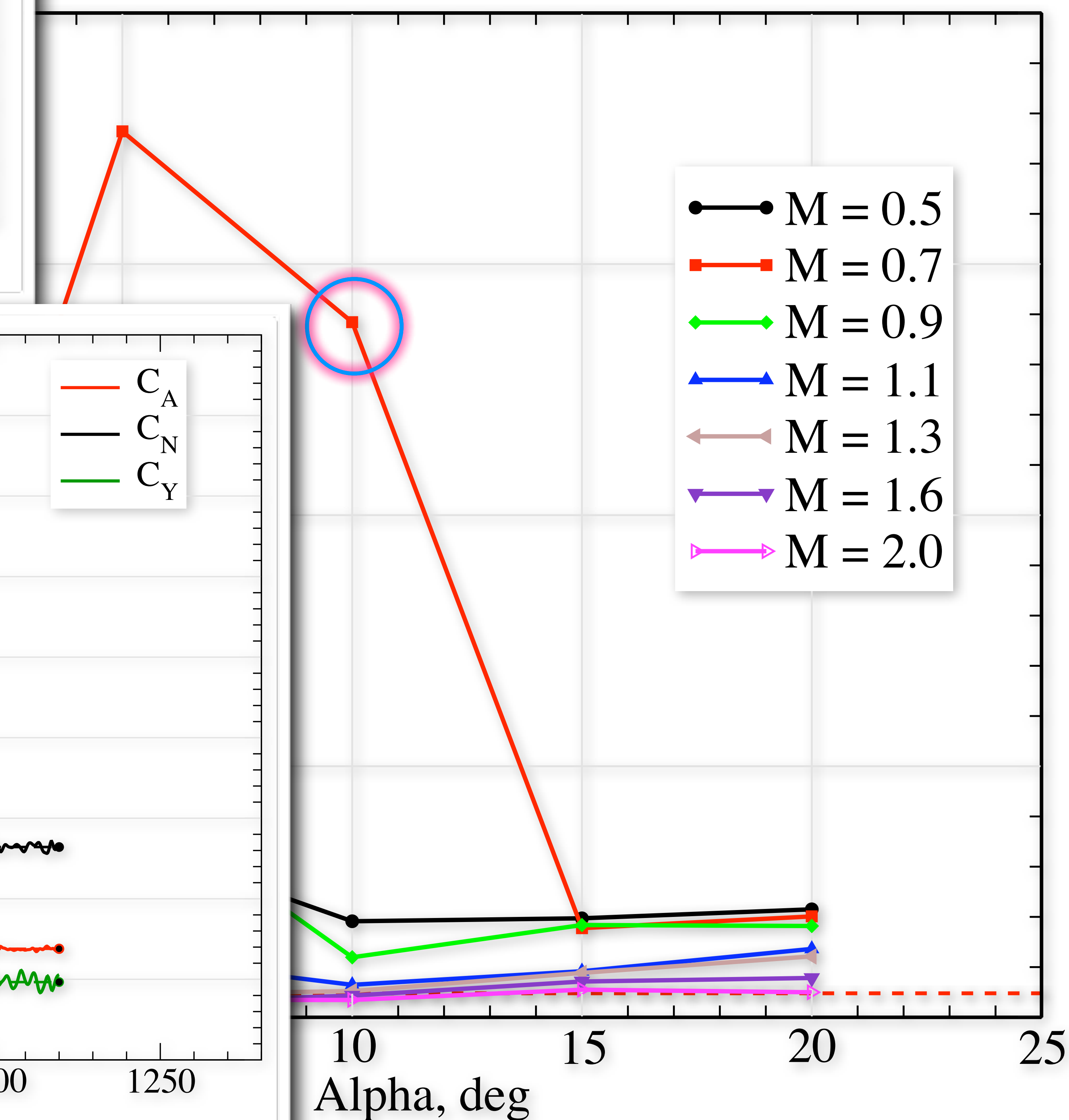
ase



Functional
Corrected Functional
Error Bound Estimate
Functional Value
Number of Cells

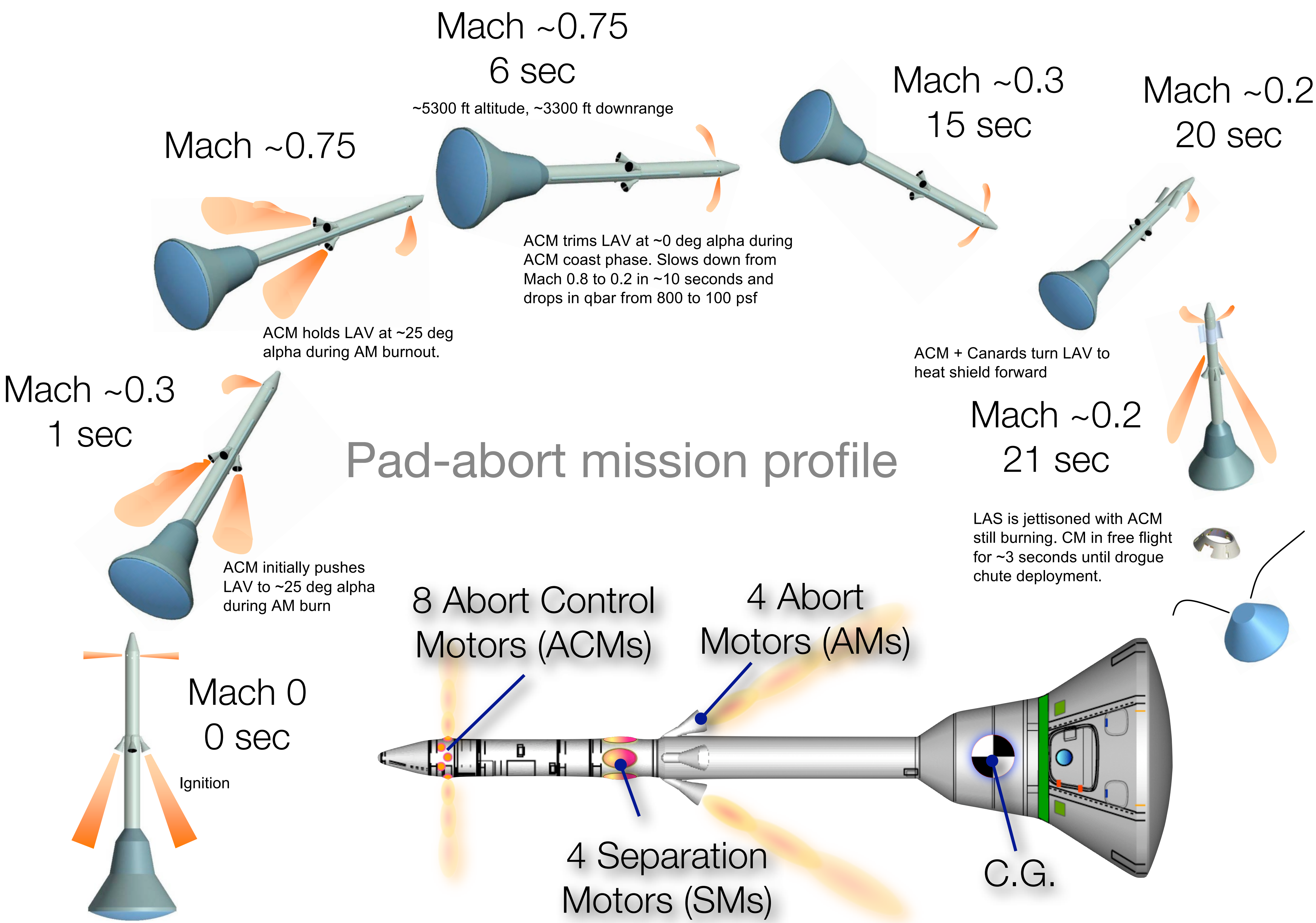


M_∞ = 0.7, α = 10°
C_A
C_N
C_Y
Force/Mom Coef.
MG Cycles



M = 0.5
M = 0.7
M = 0.9
M = 1.1
M = 1.3
M = 1.6
M = 2.0
Alpha, deg

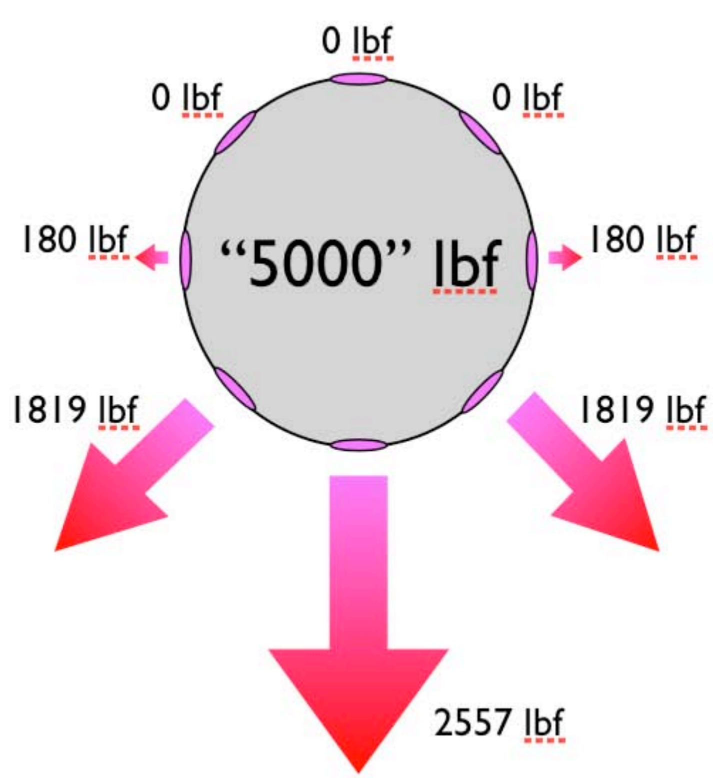
Launch Abort Vehicle with ACM Jets

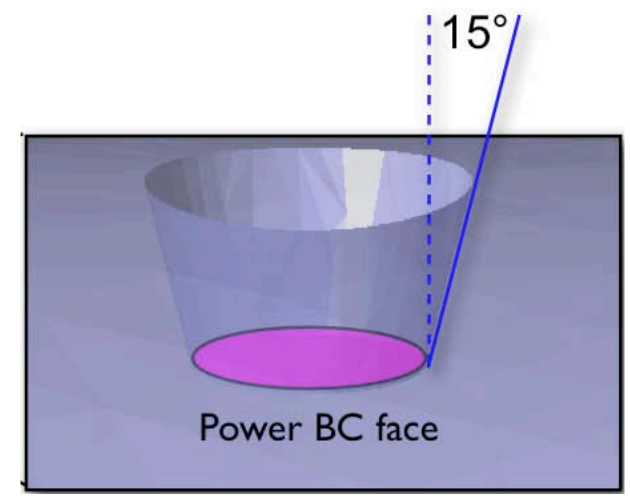


Launch Abort Vehicle with ACM Jets Problem Setup

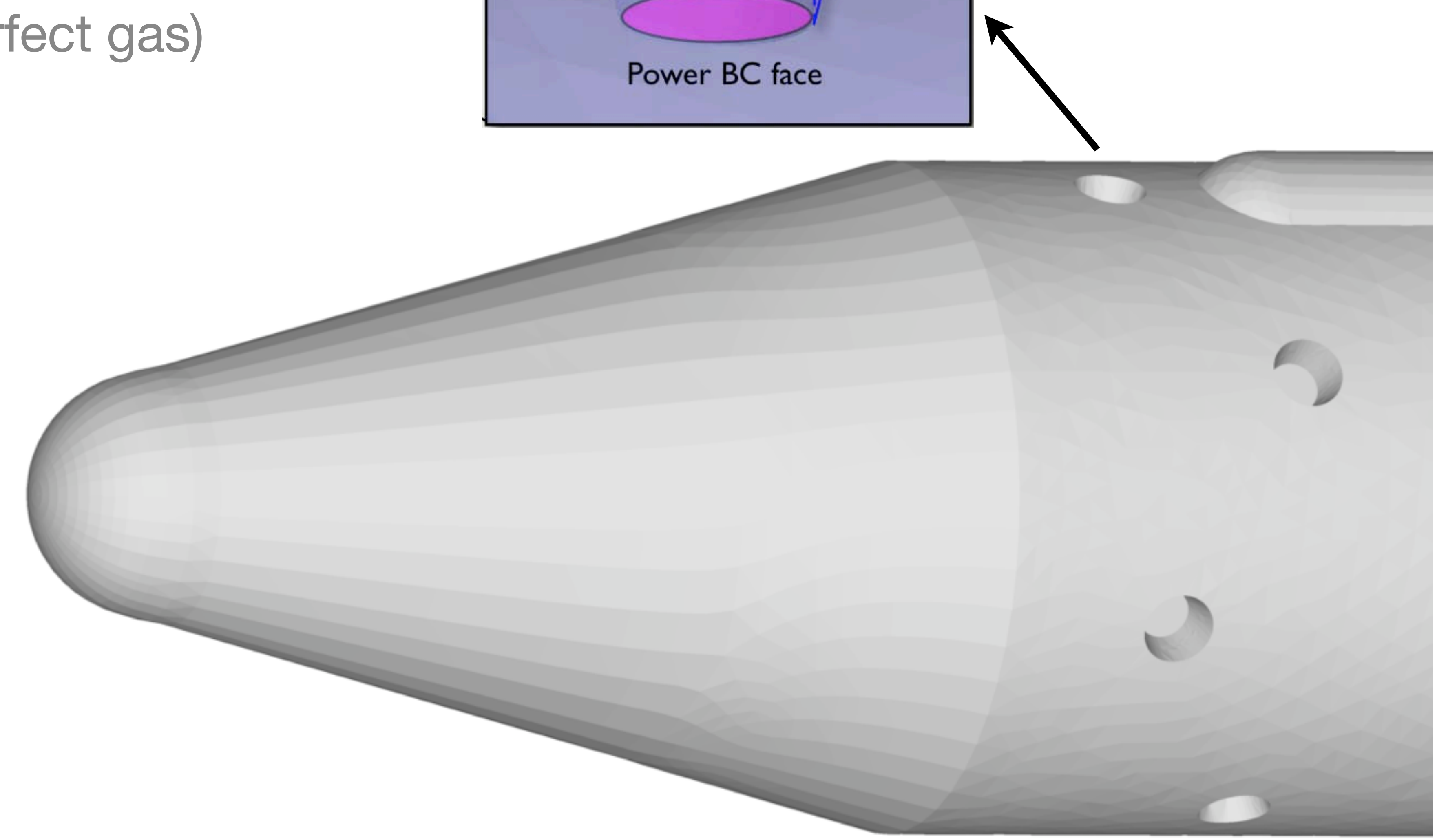
- Examine aerodynamic performance with ACM jets (AIAA 2008-1281)
- Selected case: $M_\infty = 4$, $\alpha = 20°$, due to significant plume penetration
- Power boundary conditions applied at plenum face (assumes perfect gas)
- Functional: $C_N+0.4C_A$

Thrust setting

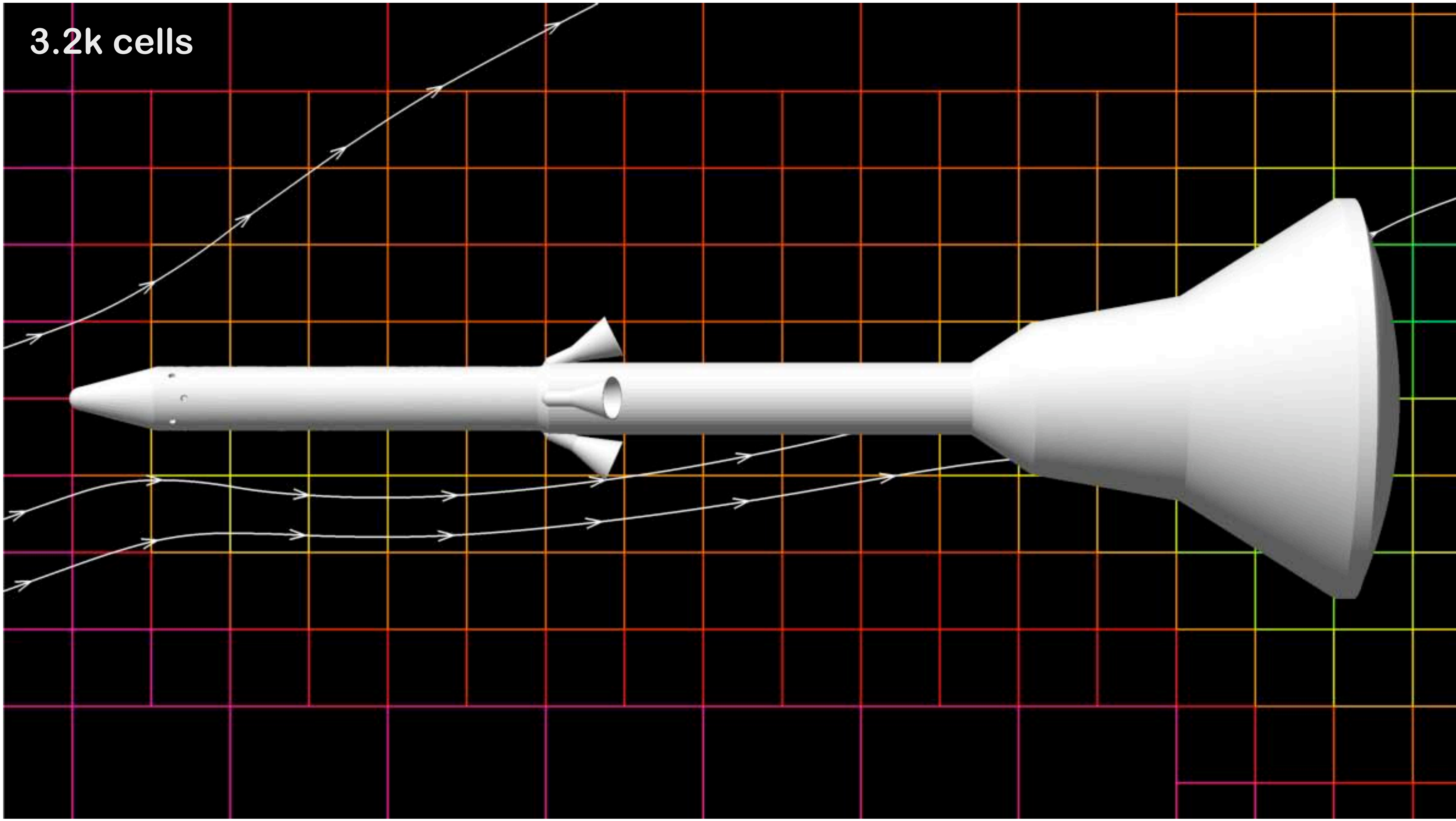




Nozzle geometry

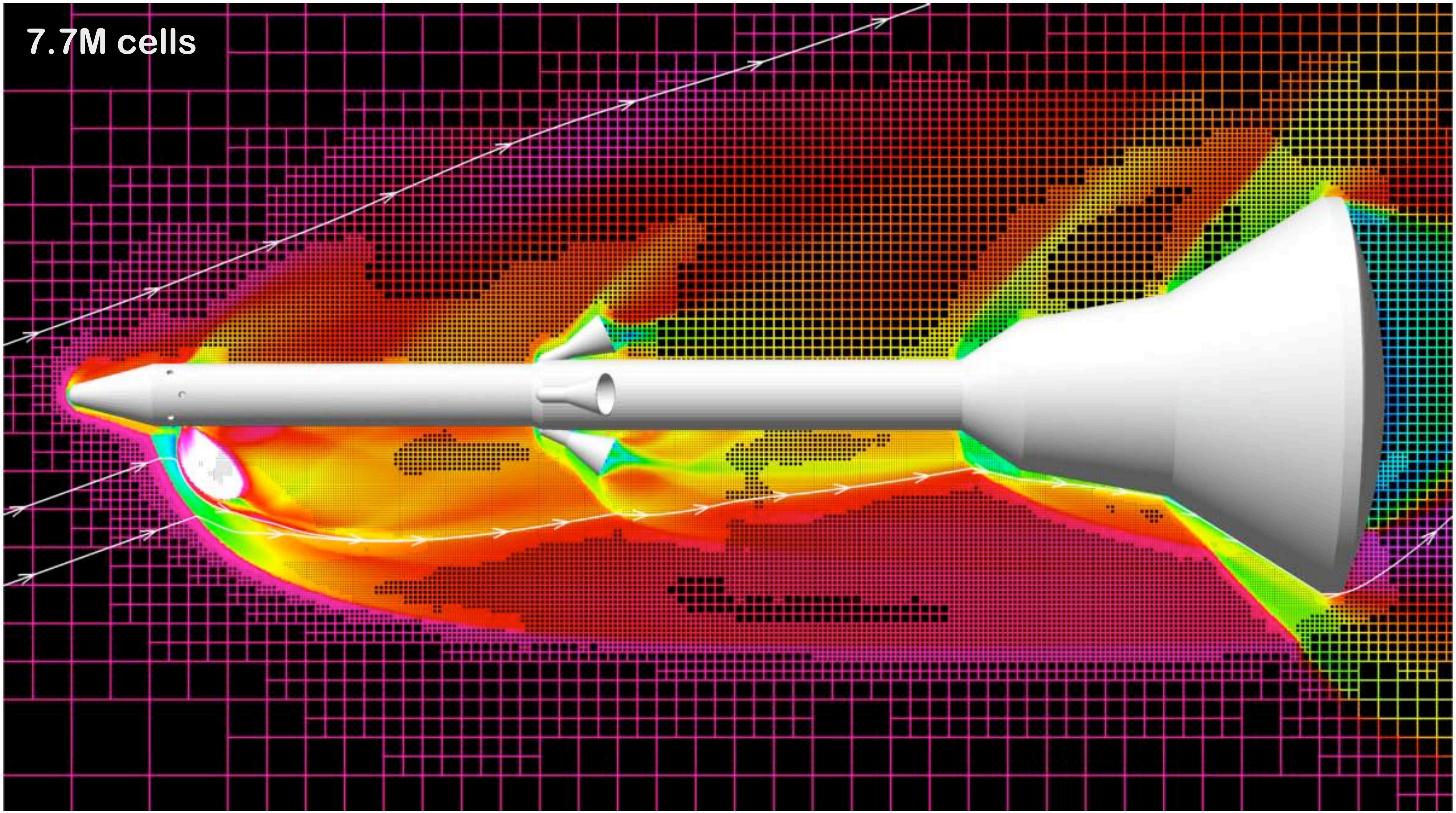


Launch Abort Vehicle
Initial mesh and solution on symmetry plane



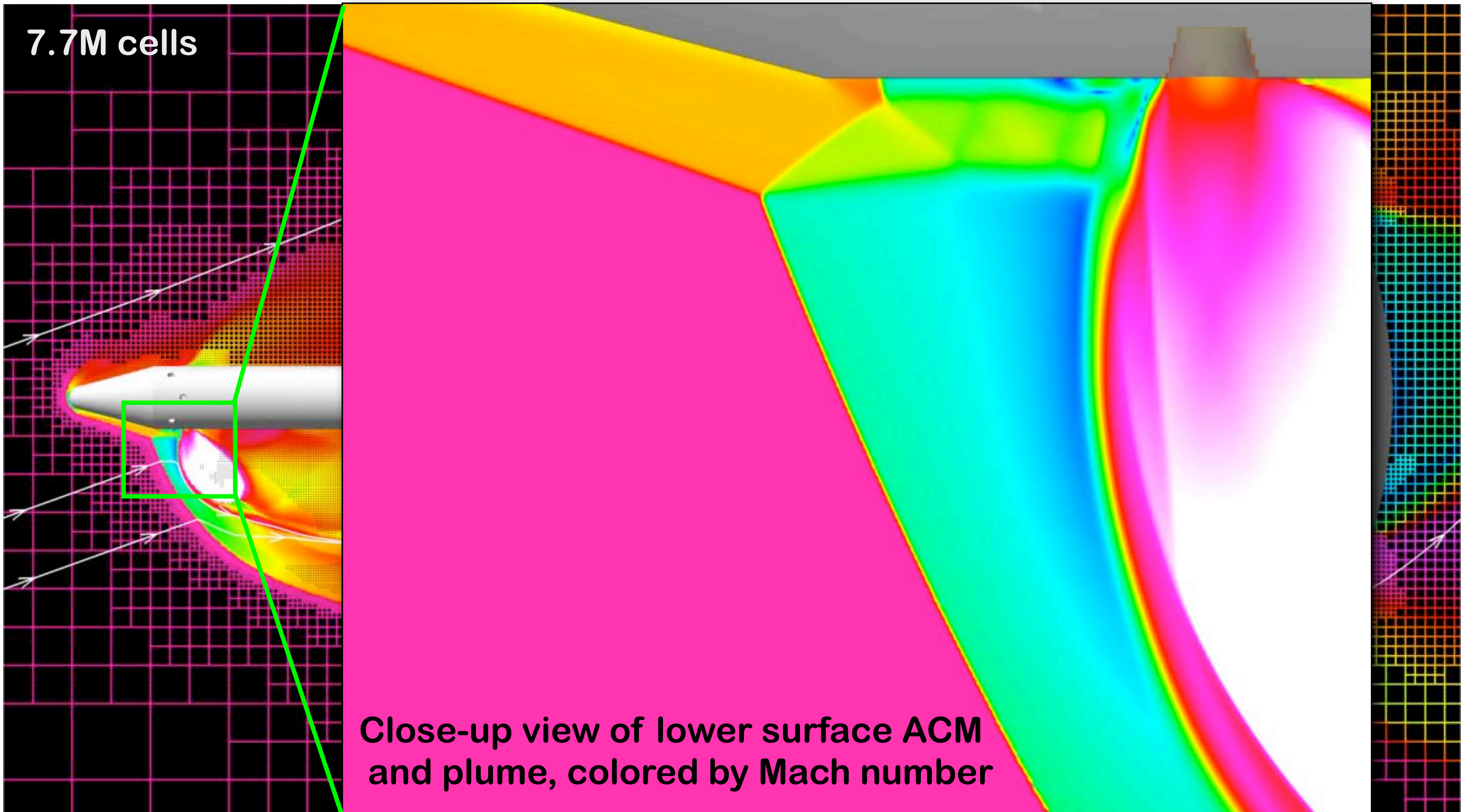
Mach contours, $M_{\infty} = 4$, $\alpha = 20°$

Launch Abort Vehicle
Final mesh and solution on symmetry plane



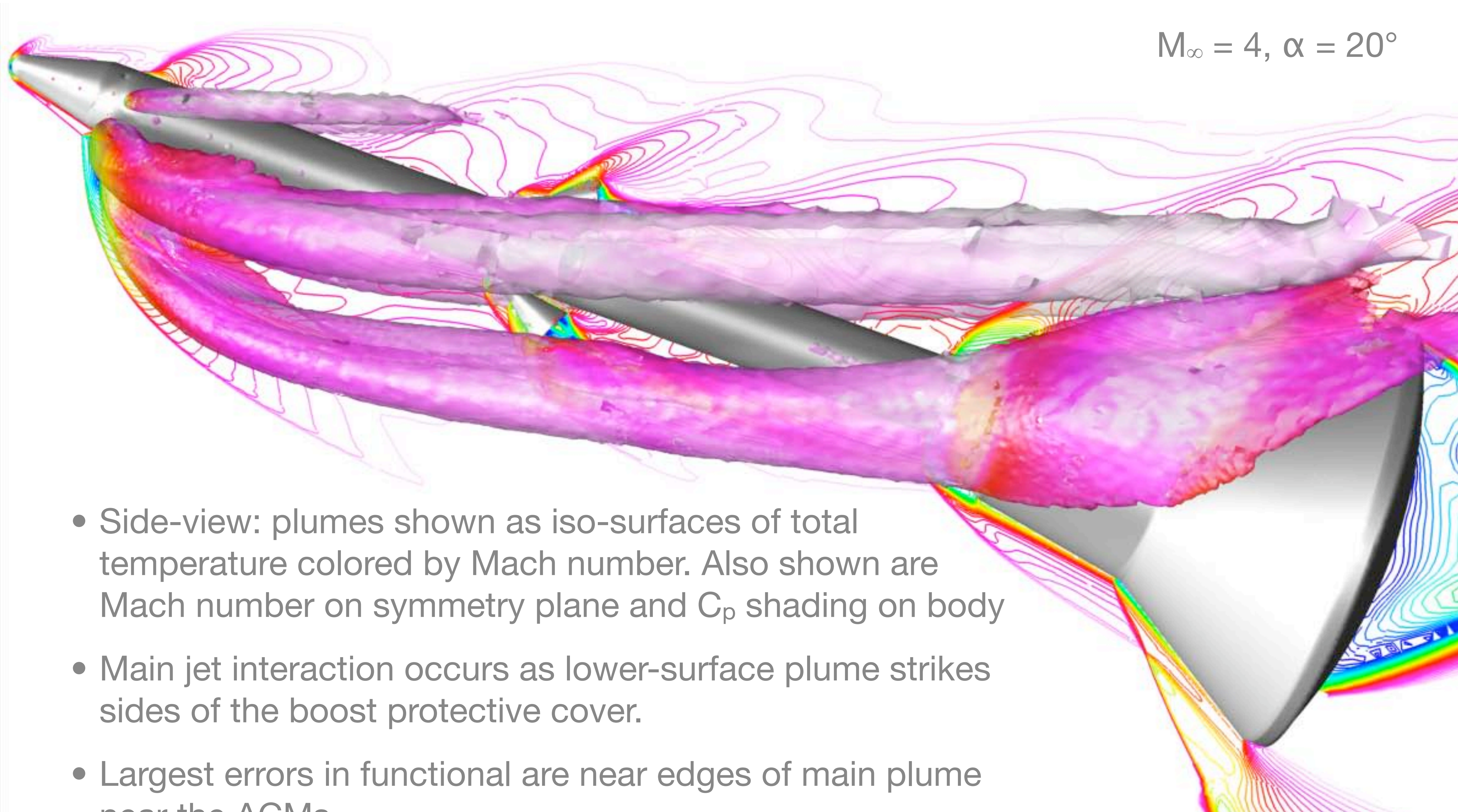
9 adaptations, Mach contours, $M_\infty = 4$, $\alpha = 20°$

Launch Abort Vehicle
Final mesh and solution on symmetry plane



9 adaptations, Mach contours, $M_\infty = 4$, $\alpha = 20°$

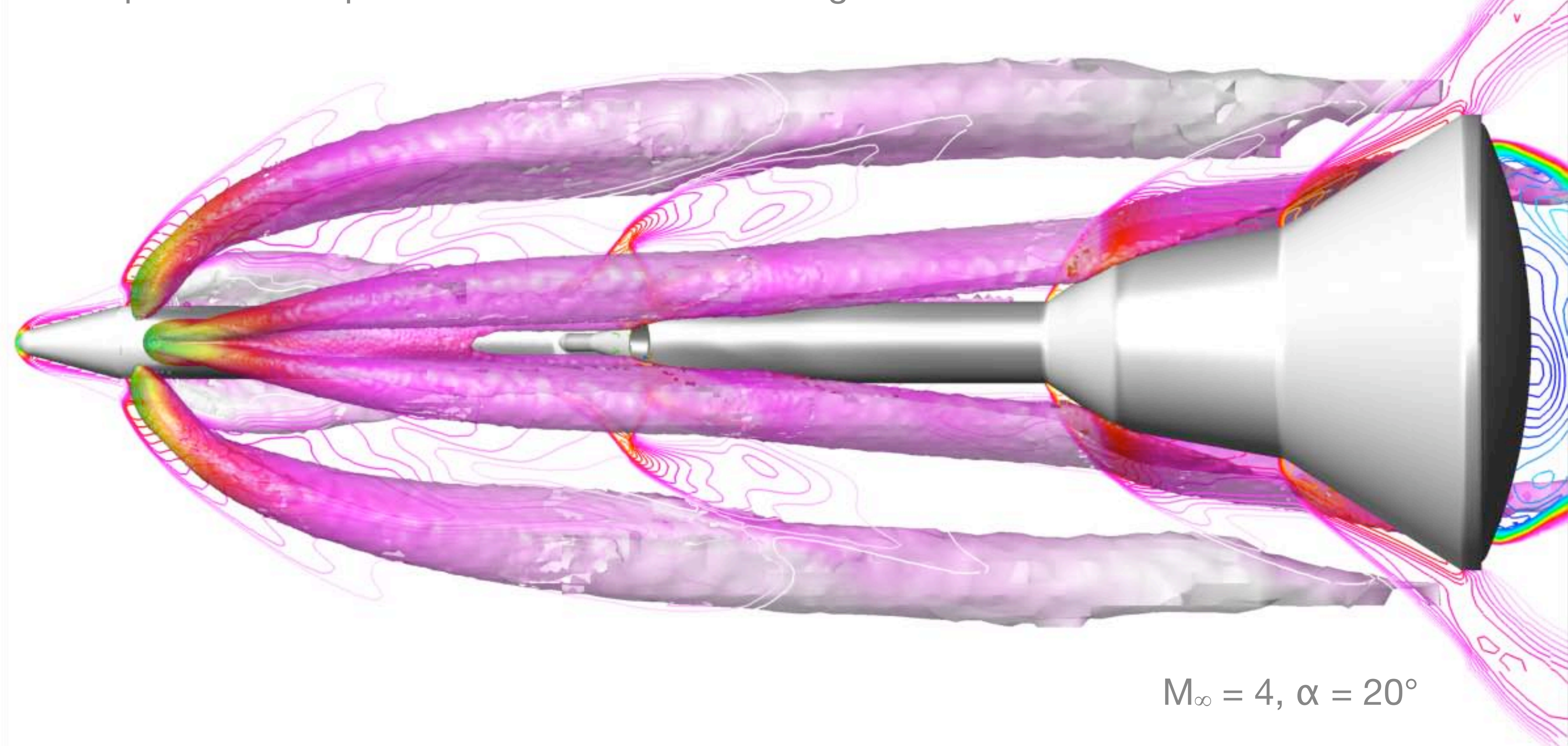
Launch Abort Vehicle
Plume Visualization on Final Mesh (~7.7M cells)



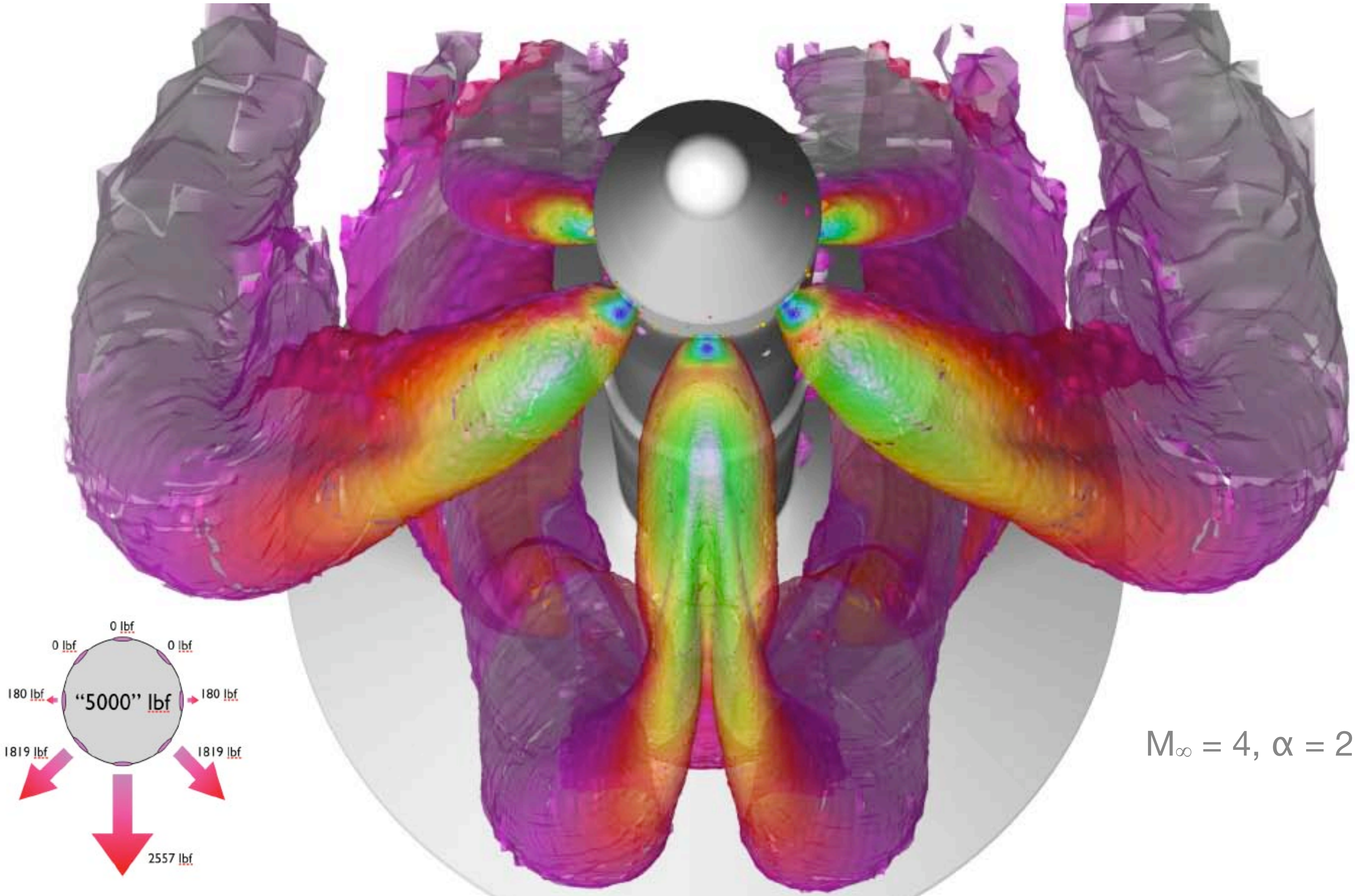
- Side-view: plumes shown as iso-surfaces of total temperature colored by Mach number. Also shown are Mach number on symmetry plane and C_p shading on body
- Main jet interaction occurs as lower-surface plume strikes sides of the boost protective cover.
- Largest errors in functional are near edges of main plume near the ACMs

Launch Abort Vehicle
Bottom view of plumes on final mesh

- Paths of three main plumes from the lower surface ACMs to the heat-shield. Main jet splits into two plumes that contact the aft region of the vehicle



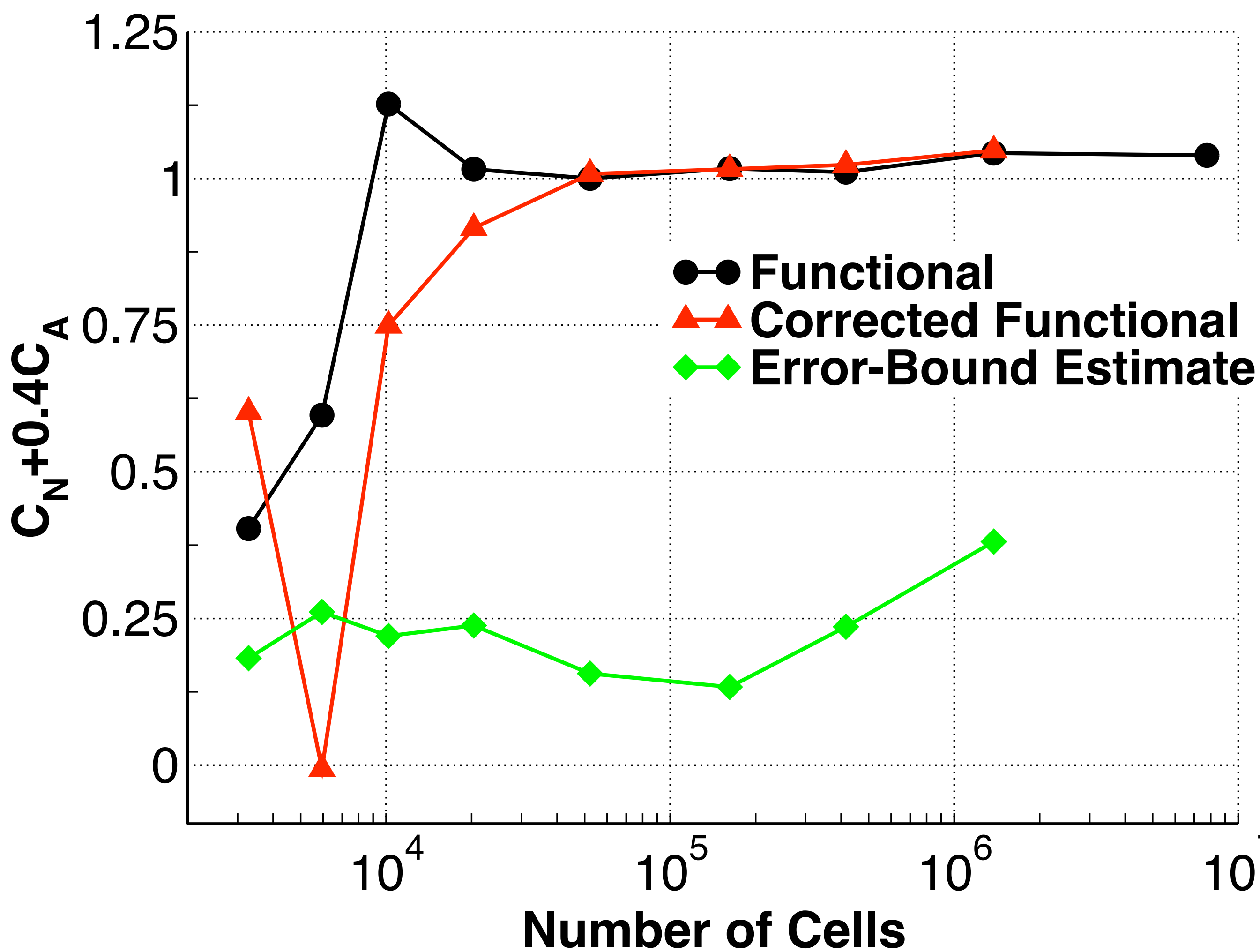
Launch Abort Vehicle
Front view of plumes on final mesh



$M_\infty = 4$, $\alpha = 20°$

Launch Abort Vehicle Functional Convergence

- Determination of plume paths and appropriate refinement of plume edges is not possible *a-priori,* yet these features determine the "aerodynamic shape" of the vehicle
- Adjoint error analysis identifies regions where jet interaction effects are important for the computation of aerodynamic coefficients and triggers mesh refinement
- Functional convergence settles down at ~1M cells, however, additional research is required to improve estimates of the error-bound



Summary

Presented a reliable and efficient approach for error estimates and mesh refinement of complex geometry problems

1. Handles complex geometry problems in an automatic fashion
2. Tolerant of coarse initial meshes
3. Behavior of functional, correction, and error estimate provide an indication of errors due to lack-of-convergence in steady simulations

➡ **It is our best mesh generator … refinement complements and surpasses expert knowledge**

➡ **Allows users to focus on data validation and analysis instead of mesh generation**

Future Work

- Sonic-boom applications (Mathias Winzter)
- Address unsteadiness issues in difficult cases
  - Affordable mesh refinement and error bound for “mildly” unsteady flow
  - Formal unsteady adjoint development
- Control accuracy of objective functions in optimization studies

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Questions?

Marian.Nemec@nasa.gov

http://people.nas.nasa.gov/~aftosmis/cart3d